1\. a) $\frac{1}{2} - \frac{x}{3} > \frac{3}{4} \quad \Big| \cdot 12$

$6 - 4x > 9$

$-4x > 3 \quad | :(-4)$

$\underline{\underline{x < -\frac{3}{4}}}$

b) $^{x)}\ \frac{1}{x} - \frac{1}{x^2} + \frac{1+x}{x^2} = \frac{x-1+1+x}{x^2} = \frac{2x}{x^2} = \underline{\underline{\frac{2}{x}}}$

c) $(6,8) \quad 3x - 5y = 11$

$-5y = 11 - 3x \quad | :(-5)$

$y = -\frac{11}{5} + \frac{3}{5}x \quad \Rightarrow k = \frac{3}{5}$

$y - 8 = \frac{3}{5}(x-6)$

$y = \frac{3}{5}x - \frac{18}{5} + 8$

$\underline{\underline{y = \frac{3}{5}x + \frac{22}{5}}} \qquad (3x - 5y + 22 = 0)$

2\. a) $f(x) = \frac{1-2x^2}{1+x^2}$

$f'(x) = \frac{-4x(1+x^2) - (1-2x^2)\cdot 2x}{(1+x^2)^2}$

$= \frac{-4x - 4x^3 - 2x + 4x^3}{(1+x^2)^2} = \underline{\underline{\frac{-6x}{(1+x^2)^2}}}$

b) $f'(x) = e^{3x} - x$

$f(x) = \int (e^{3x} - x)\,dx = \underline{\underline{\frac{1}{3}e^{3x} - \frac{1}{2}x^2 + C}}$

c) $5^n + 5^n + 5^n + 5^n + 5^n = 5^{25}$

$5 \cdot 5^n = 5^{25}$

$5^{n+1} = 5^{25}$

$n + 1 = 25$

$\underline{\underline{n = 24}}$

3\. a) $\int_0^{\pi} (1 + \sin x)\,dx = \Big/_0^{\pi} (x - \cos x) =$

$= \pi - \cos\pi - (0 - \cos 0) = \pi - (-1) + 1 = \underline{\underline{\pi + 2}}$

b) $4x^3 - 5x^2 = 2x - 3x^3$

$7x^3 - 5x^2 - 2x = 0$

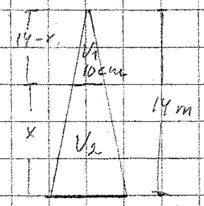
$x(7x^2 - 5x - 2) = 0$

$x = 0 \ \vee \ 7x^2 - 5x - 2 = 0$

$x = \frac{5 \pm \sqrt{25 - 4\cdot 7\cdot(-2)}}{2\cdot 7} = \frac{5 \pm 9}{14} \begin{cases} 1 \\ -\frac{2}{7} \end{cases}$

$\underline{\underline{V:\ x = 0 \ \vee\ x = 1 \ \vee\ x = -\frac{2}{7}}}$

4\.



35cm = 0,35m

Kartiot ~ $\Rightarrow$

$\frac{14-x}{14} = \frac{10}{35}$

$490 - 35x = 140$

$-35x = -350 \quad | :(-35)$

$x = 10\ (m)$

$\frac{V_1}{V} = \left(\frac{10}{35}\right)^3 \Rightarrow V_1 = \left(\frac{2}{7}\right)^3 V = \frac{8}{343}V$

$V_2 = V - \frac{8}{343}V = \frac{335}{343}V$

$2r = 0{,}35\,m \Rightarrow r = 0{,}175\,m$

$V_2 = \frac{335}{343}\cdot\frac{1}{3}\pi\cdot 0{,}175^2\cdot 14\ m^3 = 0{,}4385\ldots\ m^3$

$x \cdot 0{,}4385\ldots = 200 \quad | : 0{,}4385\ldots$

$x = 456{,}08\ldots \approx 456$

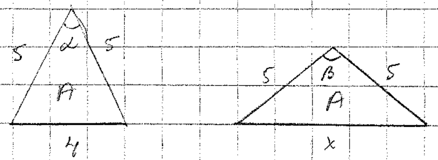
tai $V_1 = \frac{1}{3}\pi\cdot 0{,}05^2\cdot 4$

$V = \frac{1}{3}\pi\cdot 0{,}175^2\cdot 14$

$V_2 = \frac{1}{3}\pi(0{,}175^2\cdot 14 - 0{,}05^2\cdot 14) = 0{,}4385\ldots$

$\underline{\text{V: Tukin pituus 10m, kaadettiin 456 puuta}}$

5.



$$A = \frac{1}{2}\cdot 5^2 \sin\alpha = \frac{1}{2}\cdot 5^2 \sin\beta$$

$$\sin\alpha = \sin\beta$$

$$\Leftrightarrow \quad \alpha = \beta \quad \lor \quad \alpha = 180^\circ - \beta \;\Leftrightarrow\; \beta = 180^\circ - \alpha$$

$$4^2 = 5^2 + 5^2 - 2\cdot 5\cdot 5\cos\alpha$$

$$50\cos\alpha = 25 + 25 - 16$$

$$\cos\alpha = \frac{34}{50} \qquad \cos\beta = -\cos\alpha = -\frac{34}{50}$$

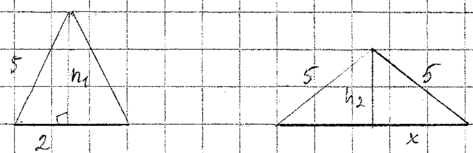
$$x^2 = 5^2 + 5^2 - 2\cdot 5\cdot 5\cdot\left(-\frac{34}{50}\right)$$

$$x^2 = 25 + 25 + 34$$

$$x^2 = 84$$

$$x = \pm\sqrt{84} = \underline{\underline{2\sqrt{21} \approx 9{,}2}}$$

tai



$$h_1^2 + 2^2 = 5^2 \qquad h_2^2 + x^2 = 5^2$$

$$h_1 = \sqrt{21} \qquad h_2 = \sqrt{25 - x^2}$$

$$A = \frac{1}{2}\cdot 4\cdot\sqrt{21} = \frac{1}{2}\cdot 2x\sqrt{25 - x^2}$$

$$2\sqrt{21} = x\sqrt{25 - x^2} \quad |\,(\;)^2 \text{ mol. } \geq 0$$

$$84 = x^2(25 - x^2)$$

$$84 = 25x^2 - x^4$$

$$x^4 - 25x^2 + 84 = 0 \quad |\; x^2 = t$$

$$t^2 - 25t + 84 = 0$$

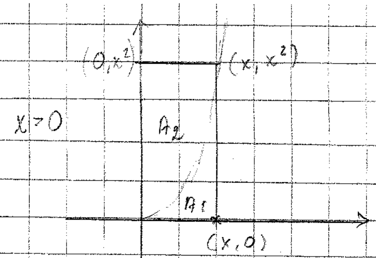
$$t = \frac{25 \pm \sqrt{625 - 4\cdot 1\cdot 84}}{2\cdot 1} = \frac{25 \pm 17}{2} \begin{cases} 21 \\ 4 \end{cases}$$

$$x^2 = 21 \quad \lor \quad x^2 = 4$$

$x = \sqrt{21} \quad \lor \quad x = 2$ (annettu kolmio)

Kanta $2x = \underline{\underline{2\sqrt{21} \approx 9{,}2}}$

6.



$$A = x\cdot x^2 = x^3$$

$$A_1 = \int_0^x t^2\,dt = \Big/_0^x \frac{1}{3}t^3 = \frac{x^3}{3}$$

$$A_2 = A - A_1 = x^3 - \frac{x^3}{3} = \frac{2}{3}x^3$$

$$\frac{A_1}{A_2} = \frac{\frac{1}{3}x^3}{\frac{2}{3}x^3} = \frac{1}{2}$$

$\underline{\underline{V: 1:2}}$

7. $\sqrt{2-x} = x + 2$ määr. ehto $2 - x \geq 0$

$x \leq 2.$

$\sqrt{2-x} = x + 2 \;|\,(\;)^2$ ehto: $x + 2 \geq 0$

$x \geq -2$

$$\boxed{-2 \leq x \leq 2}$$

$$2 - x = x^2 + 4x + 4$$

$$x^2 + 5x + 2 = 0$$

$$x = \frac{-5 \pm \sqrt{25 - 4\cdot 1\cdot 2}}{2\cdot 1} = \frac{-5 \pm \sqrt{17}}{2} \begin{cases} -0{,}438\ldots \\ -4{,}56\ldots \text{ ei käy} \end{cases}$$

$$\underline{\underline{V: x = \frac{-5 + \sqrt{17}}{2}}}$$

8. 2V 3M nostetaan 2 x = mustia

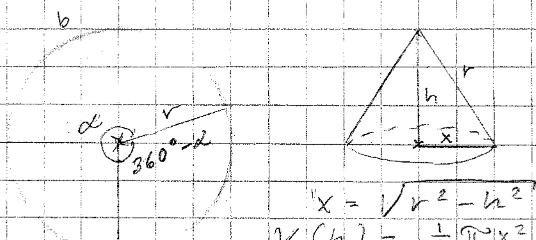
$$P(x=0) = \frac{2\cdot 1}{5\cdot 4} = \underline{\underline{\frac{1}{10}}}$$

$$P(x=1) = \frac{\binom{2}{1}\binom{3}{1}}{\binom{5}{2}} = \frac{6}{10} = \underline{\underline{\frac{3}{5}}}$$

$$P(x=2) = \frac{3\cdot 2}{5\cdot 4} = \underline{\underline{\frac{3}{10}}}$$

$$E(\text{mustia}) = \frac{1}{10}\cdot 0 + \frac{6}{10}\cdot 1 + \frac{3}{10}\cdot 2 = \frac{12}{10} = \underline{\underline{1{,}2}}$$

9.



$x = \sqrt{r^2 - h^2}$ $\qquad 0 \le h \le r$

$V(h) = \frac{1}{3}\pi x^2 h = \frac{1}{3}\pi(r^2 - h^2)h$

$= \frac{\pi}{3}(r^2h - h^3)$ jatk., deriv.

$V'(h) = \frac{\pi}{3}(r^2 - 3h^2) = 0$

$-3h^2 = r^2$

$h^2 = \frac{r^2}{3}$

$h = \pm\frac{r}{\sqrt{3}}$

	$-\frac{r}{\sqrt{3}}$		0		$\frac{r}{\sqrt{3}}$		r
$V'(h)$	−	+		+ +		−	
$V(h)$				↗		↘	

Tilavuus suurin, kun $h = \frac{r}{\sqrt{3}}$

tällöin $x = \sqrt{r^2 - \frac{r^2}{3}} = r\sqrt{\frac{2}{3}}$

$b = 2\pi x = 2\pi r\sqrt{\frac{2}{3}}$

$\frac{\alpha}{360°} = \frac{2\pi r\sqrt{\frac{2}{3}}}{2\pi r} \Rightarrow \alpha = \sqrt{\frac{2}{3}}\cdot 360°$

$= 293{,}93\ldots°$

$360° - \alpha \approx \underline{\underline{66°}}$

10. $x \in \mathbb{R}$

V: $(1-x)^8 \ge 1 - 8x$

Tod: $(1-x)^8 \ge 1 - 8x \Leftrightarrow f(x) = (1-x)^8 + 8x - 1 \ge 0$

jatk., deriv.

$f'(x) = -8(1-x)^7 + 8 = 0$

$-8(1-x)^7 = -8 \quad |:(-8)$ $\qquad f'(-1) = -1016$

$(1-x)^7 = 1 \quad |\sqrt[7]{\ }$ $\qquad f'(1) = 8$

$1 - x = 1$

$x = 0$

		0	
$f'(x)$	− −		+ + +
$f(x)$	↘		↗

Pienin arvo

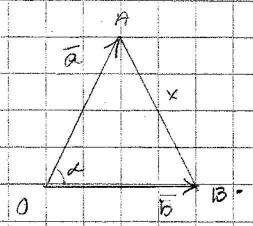
$f(0) = (1-0)^8 + 8\cdot 0 - 1 = 0$

$\Rightarrow f(x) \ge 0 \quad \forall x \in \mathbb{R} \Rightarrow$ väite □

S08 MAA

11. $\overline{OA} = \bar{a}\ (\ne \bar{0})$ $\quad \overline{OB} = \bar{b}$

$\bar{a}\cdot\bar{a} = 2\bar{a}\cdot\bar{b} \quad \Leftrightarrow \quad |\bar{a}|^2 = 2|\bar{a}||\bar{b}|\cos\alpha$



$x^2 = |\bar{a}|^2 + |\bar{b}|^2 - 2|\bar{a}||\bar{b}|\cos\alpha$

$x^2 = |\bar{a}|^2 + |\bar{b}|^2 - |\bar{a}|^2$

$x^2 = |\bar{b}|^2$

$x = \pm|\bar{b}|$

$\Leftrightarrow |\overline{OB}| = |\overline{AB}|$ ja kolmio on tasakylkinen □

($|\bar{a}|^2 = 2|\bar{a}||\bar{b}|\cos\alpha \quad |:|\bar{a}| \ne 0$

$|\bar{a}| = 2|\bar{b}|\cos\alpha = |\bar{b}|$, jos $\cos\alpha = \frac{1}{2}$ eli $\alpha = 60°$

tällöin kolmio on tasasivuinen!)

12. $f(x) = \frac{3x^3 - x^2 - 12x + a}{x + 2}$, $x \ne -2$

Funktiolla on raja-arvo kohdassa $x = -2$, jos $x + 2$ supistuu pois eli $x = -2$ on myös osoittajan nollakohta

$3\cdot(-2)^3 + (-2)^2 - 12\cdot(-2) + a = 0$

$-24 - 4 + 24 + a = 0 \Leftrightarrow \underline{\underline{a = 4}}$

$3x^3 - x^2 - 12x + 4 = x^2(3x - 1) - 4(3x - 1)$

$= (x^2 - 4)(3x - 1) = (x + 2)(x - 2)(3x - 1)$

$\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{\cancel{(x+2)}(x-2)(3x-1)}{\cancel{x+2}} = -4\cdot(-7)$

$= \underline{\underline{28}}$

13. $f(x) = x^3$

$$\frac{f(x+h)-f(x-h)}{2h} = \frac{(x+h)^3-(x-h)^3}{2h}$$

$$= \frac{x^3+3x^2h+3xh^2+h^3-(x^3-3x^2h+3xh^2-h^3)}{2h}$$

$$= \frac{\cancel{x^3}+3x^2h+\cancel{3xh^2}+h^3-\cancel{x^3}+3x^2h-\cancel{3xh^2}+h^3}{2h}$$

$$= \frac{2h(3x^2+h^2)}{2h} = \underline{\underline{3x^2+h^2}}$$

$$\frac{f(x+h)-f(x-h)}{2h} = \frac{f(x+h)-f(x)+f(x)-f(x-h)}{2h}$$

$$= \frac{1}{2}\cdot\frac{f(x+h)-f(x)}{h} + \frac{1}{2}\cdot\frac{f(x)-f(x-h)}{h}$$

$$= \frac{1}{2}\left[\frac{f(x+h)-f(x)}{h} + \frac{f(x-h)-f(x)}{-h}\right] \xrightarrow{h\to 0} \frac{1}{2}\cdot 2f'(x) = \underline{\underline{f'(x)}} \quad \square$$

$$h \to 0 \Rightarrow \frac{f(x+h)-f(x)}{h} \to f'(x)$$

$$\Rightarrow -h \to 0 \Rightarrow \frac{f(x-h)-f(x)}{-h} \to f'(x)$$

14. $|x| = \begin{cases} x, \text{ kun } x \geq 0 \\ -x, \text{ kun } x < 0 \end{cases}$

a) $x \leq |x| \Leftrightarrow x - |x| \leq 0$

$x \geq 0 \quad x - |x| = x - x = 0$ tosi

$x < 0 \quad x - |x| = x - (-x) = 2x < 0$ tosi

b) $x + y \overset{a)}{\leq} |x| + y \overset{a)}{\leq} |x| + |y| \quad \square$

c) V: $|x+y| \leq |x| + |y|$

$x+y \geq 0 \quad |x+y| = x+y \leq |x| + |y| \quad \square$

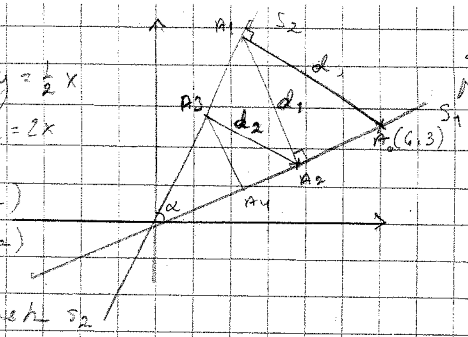
$x+y < 0 \quad |x+y| = -x-y \leq |-x| + |-y| = |x| + |y| \quad \square$

d) V: $\big||x|-|y|\big| \leq |x| + |y|$

$$\big||x|-|y|\big| = \big||x|+(-|y|)\big| \overset{c)}{\leq} \big||x|\big| + \big|-|y|\big| = |x| + |y| \quad \square$$

15. $s_1: \ x - 2y = 0 \Leftrightarrow y = \frac{1}{2}x$

$s_2: \ 2x - y = 0 \Leftrightarrow y = 2x$



Kolmiot OA_nA_{n+1} ~ (kk)

($\sphericalangle \alpha$ yht., suora kulma)

pisteen A et. suorasta s_2

$$d = \frac{|2\cdot 6 - 3|}{\sqrt{2^2+1^2}} = \frac{9}{\sqrt{5}}$$

$k_2 = 2$

$k_{AA_1} = -\frac{1}{2} \quad (-\frac{1}{2}\cdot 2 = -1)$

$y - 3 = -\frac{1}{2}(x-6)$

$\begin{cases} y = -\frac{1}{2}x + 6 \\ y = 2x \end{cases} \qquad 2x = -\frac{1}{2}x + 6, \quad \frac{5}{2}x = 6, \quad x = \frac{12}{5} \Rightarrow y = \frac{24}{5}$

$A_1(\frac{12}{5}, \frac{24}{5})$

A_1:n et. suorasta s_1

$$d_1 = \frac{|\frac{12}{5} - 2\cdot\frac{24}{5}|}{\sqrt{2^2+1^2}} = \frac{36}{5\sqrt{5}}$$

$k_1 = \frac{1}{2} \qquad y - \frac{24}{5} = -2(x - \frac{12}{5})$

$k_{A_1A_2} = -2 \qquad \begin{cases} y = -2x + \frac{48}{5} \\ y = \frac{1}{2}x \end{cases} \qquad \frac{1}{2}x = -2x + \frac{48}{5}, \quad \frac{5}{2}x = \frac{48}{5}, \quad x = \frac{96}{25}, \quad y = \frac{1}{2}x = \frac{48}{25}$

$A_2 = (\frac{96}{25}, \frac{48}{25})$

$$d_2 = \frac{|2\cdot\frac{96}{25} - \frac{48}{25}|}{\sqrt{2^2+1^2}} = \frac{144}{25\sqrt{5}}$$

$\frac{9}{\sqrt{5}}, \frac{36}{5\sqrt{5}}, \frac{144}{25\sqrt{5}}, \ldots$ muod. geom. jonon*

$$\frac{d_1}{d} = \frac{\frac{36}{5\sqrt{5}}}{\frac{9}{\sqrt{5}}} = \frac{4}{5} \qquad \frac{d_2}{d_1} = \frac{\frac{144}{25\sqrt{5}}}{\frac{36}{5\sqrt{5}}} = \frac{4}{5}$$

$$S = \frac{d}{1-q} = \frac{\frac{9}{\sqrt{5}}}{1-\frac{4}{5}} = \frac{45}{\sqrt{5}} = \frac{45\sqrt{5}}{5} = \underline{\underline{9\sqrt{5} \approx 20{,}1}}$$

* 15. Tod:

Merk $A = A_0$

$n = 0, 2, 4, \ldots$

leikkauspiste A_{n+1} löytyy:

$A_n = (x_n, \frac{1}{2} x_n)$ (suoralla s_1 $y = \frac{1}{2}x$)

suoran s_2 $y = 2x$ normaalin $k = -\frac{1}{2}$

ja yhtälö $y - \frac{1}{2} x_n = -\frac{1}{2}(x - x_n)$

leikk. piste $\begin{cases} y = -\frac{1}{2}x + x_n \\ y = 2x \end{cases}$

$$2x = -\frac{1}{2}x + x_n$$

$$\frac{5}{2}x = x_n \Rightarrow x_{n+1} = \frac{2}{5} x_n$$

$$y_{n+1} = 2 \cdot \frac{2}{5} x_n = \frac{4}{5} x_n$$

$$d_n = A_n A_{n+1} = \frac{|2 x_n - \frac{1}{2} x_n|}{\sqrt{5}} = \frac{3 x_n}{2\sqrt{5}} \quad (A_n \to s_2)$$

A_{n+1} et. s_1:stä

$$d_{n+1} = A_{n+1} A_{n+2} = \frac{|\frac{2}{5} x_n - 2 \cdot \frac{4}{5} x_n|}{\sqrt{5}} = \frac{6 x_n}{5\sqrt{5}}$$

$$\frac{d_{n+1}}{d_n} = \frac{\frac{6 x_n}{5\sqrt{5}}}{\frac{3 x_n}{2\sqrt{5}}} = \frac{4}{5} = q$$ geom. jono!