

1. a)  $(x-2)(x-3) = 6$   
 $x^2 - 3x - 2x + 6 = 6$   
 $x^2 - 5x = 0$   
 $x(x-5) = 0$   
 $x = 0 \vee x = 5$

b) x)  $\frac{x-b}{x-3} - \frac{1}{x} = 1$   $x \neq 0 \wedge x \neq 3$   
 $\frac{x^2 - x + 3}{x(x-3)} = \frac{x^2 - 3x}{x(x-3)}$   
 $x^2 - x + 3 = x^2 - 3x$   
 $2x = -3$   
 $x = -1\frac{1}{2}$

c)  $\sqrt{27-10\sqrt{2}} = 5-\sqrt{2}$   $\wedge$   $\sqrt{27-10\sqrt{2}} > 0 \wedge 5-\sqrt{2} > 0 \wedge (5-\sqrt{2})^2 = 27-10\sqrt{2}$   
 $1^\circ 27-10\sqrt{2} = 12,85... > 0$   
 $2^\circ 5-\sqrt{2} = 3,58... > 0$   
 $3^\circ (5-\sqrt{2})^2 = 25-10\sqrt{2}+2 = 27-10\sqrt{2} \Rightarrow \sqrt{27-10\sqrt{2}} = 5-\sqrt{2} \quad \square$

2. a)  $6(x-1) + 4 \geq 3(2x+1)$   
 $6x-6+4 \geq 21x+3$   
 $-15x \geq 5 \quad | :(-15)$   
 $x \leq -\frac{1}{3}$

b)  $\sqrt{x+2} = 3/(\cdot)^2$   $x+2 \geq 0 \Leftrightarrow x \geq -2$   $\wedge$   $\text{mol. puolet} \geq 0$   
 $x+2 = 9$   
 $x = 7$

c)  $\sin \frac{x}{2} = \frac{1}{\sqrt{2}}$   
 $\frac{x}{2} = \frac{\pi}{4} + n \cdot 2\pi \vee \frac{x}{2} = \frac{3\pi}{4} + n \cdot 2\pi$   
 $x = \frac{\pi}{2} + n \cdot 4\pi \vee x = \frac{3\pi}{2} + n \cdot 4\pi \quad n \in \mathbb{Z}$

3. a)  $f(x) = ax^2e^x + bxe^x$   $f'(x) = 2x^2e^x + xe^x - 3e^x$   
 $f'(x) = 2axe^x + ax^2e^x + be^x + bxe^x$   
 $= e^x(ax^2 + (2a+b)x + b) \equiv e^x(2x^2 + x - 3)$   
 $\begin{cases} a = 2 \\ 2a+b = 1 \\ b = -3 \end{cases} \quad 2 \cdot 2 - 3 = 1 \quad \underline{a = 2 \wedge b = -3}$

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3b)  $\int_{-1}^{\frac{4}{9}} \frac{1}{\sqrt{x+5}} dx = \int_{-1}^{\frac{4}{9}} (x+5)^{-\frac{1}{2}} dx = \int_{-1}^{\frac{4}{9}} 2(x+5)^{\frac{1}{2}} dx$   
 $= 2\sqrt{\frac{4}{9}+5} - 2\sqrt{-1+5} = 2\sqrt{\frac{49}{9}} - 2\sqrt{4} = 2 \cdot \frac{7}{3} - 4 = \underline{\underline{\frac{2}{3}}}$

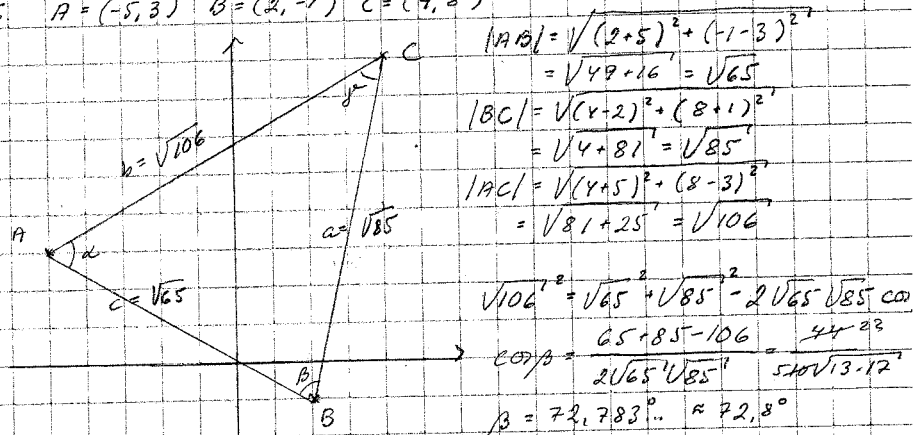
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4. Sivut  $2x, 3x, 4x$   $V = 10l = 10 \text{ dm}^3$   
 $2x \cdot 3x \cdot 4x = 10$   
 $24x^3 = 10 \quad | :24$   
 $x^3 = \frac{5}{12} \quad | \sqrt[3]{\quad}$   
 $x = \sqrt[3]{\frac{5}{12}} (= 0,7469... \text{ dm})$   
 $2x = 1,4938... \text{ dm} \approx 14,9 \text{ cm}$   
 $3x = 2,2407... \text{ dm} \approx 22,4 \text{ cm}$   
 $4x = 2,9876... \text{ dm} \approx 29,9 \text{ cm}$   $V: 14,9 \text{ cm}, 22,4 \text{ cm}, \text{ ja } 29,9 \text{ cm}$

5.  $A = (2, 3, 1)$   $B = (-5, 7, 2)$   $C = (1, 5, 3)$   
 $\overline{AB} + \overline{BC} + \overline{CA} = \overline{AA} = \underline{\underline{0}}$

$\overline{AB} = (-5-2)\vec{i} + (7-3)\vec{j} + (2-1)\vec{k} = -7\vec{i} + 4\vec{j} + \vec{k}$   
 $\overline{BC} = (1+5)\vec{i} + (5-7)\vec{j} + (3-2)\vec{k} = 6\vec{i} - 2\vec{j} + \vec{k}$   
 $\overline{CA} = (2-1)\vec{i} + (3-5)\vec{j} + (1-3)\vec{k} = \vec{i} - 2\vec{j} - 2\vec{k}$   
 $\overline{AB} + \overline{BC} + \overline{CA} = (-7+6+1)\vec{i} + (4-2-2)\vec{j} + (1+1-2)\vec{k} = \underline{\underline{0}}$

6.  $A = (-5, 3)$   $B = (2, -1)$   $C = (4, 8)$



$\sqrt{85}^2 = \sqrt{65}^2 + \sqrt{106}^2 - 2\sqrt{65}\sqrt{106} \cos \alpha$   
 $\cos \alpha = \frac{65+106-85}{2\sqrt{65}\sqrt{106}} = \frac{86}{2\sqrt{65}\sqrt{106}}$   $V: 72,8^\circ, 58,8^\circ \text{ ja } 48,4^\circ$   
 $\alpha = 58,799... \approx 58,8^\circ$   
 $\gamma = 180^\circ - \alpha - \beta \approx 48,4^\circ$

6b)  $A = \frac{1}{2} bc \sin \alpha = \frac{1}{2} \cdot \sqrt{106} \cdot \sqrt{65} \cdot \sin 58,799...^\circ = 35,49... \text{ MAA}$   
 $\approx 35,5$  509

7. tilanne x p E (vakuumäärä) =  $\frac{1}{4} \cdot (-150) + \frac{3}{4} \cdot 75$   
 $= \frac{75}{4} = 18,75$   
V: 18,75 €

4. lla joll	-3:60	$\frac{1}{4}$
ei -"	+3:25	$\frac{3}{4}$

8.  $-x^2 + x + 2 > 0$   $x^3 + 2x^2 - 3x \neq 0$   
 $x^3 + 2x^2 - 3x \neq 0$   
 $x(x^2 + 2x - 3) \neq 0$   
 $x \neq 0 \vee x \neq \frac{-2 \pm \sqrt{4 - 4 \cdot (-3)}}{2 \cdot 1} = \frac{-2 \pm 4}{2} = -3$

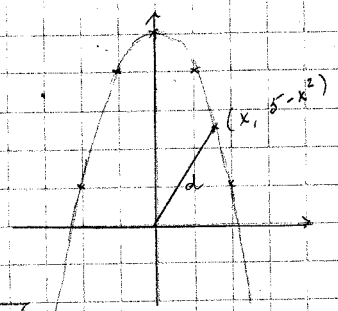
$(x^2 - x - 2) \neq 0$   
 $x = \frac{1 \pm \sqrt{1 - 4 \cdot (-2)}}{2 \cdot 1} = \frac{1 \pm 3}{2} = -1$

$-x^2 + x + 2$	-	+	+	-
x	-	-	+	+
$x^2 + 2x - 3$	+	-	-	+
osau.	+	-	+	-

V:  $x < -3 \vee -1 < x < 0 \vee 1 < x < 2$

9.  $y = 5 - x^2$

x	4
	-2
	-1
huippu	0
	1
	2



$d(x) = \sqrt{x^2 + (5 - x^2)^2}$   
 $= \sqrt{x^2 + 25 - 10x^2 + x^4} = \sqrt{x^4 - 9x^2 + 25}$

jalki, deriva  
 $d'(x) = \frac{1}{2} (x^4 - 9x^2 + 25)^{-\frac{1}{2}} \cdot (4x^3 - 18x)$   
 $= \frac{2x(2x^2 - 9)}{2\sqrt{x^4 - 9x^2 + 25}} = 0$   
 $x = 0 \vee 2x^2 - 9 = 0$   
 $x = \pm \frac{3}{\sqrt{2}}$

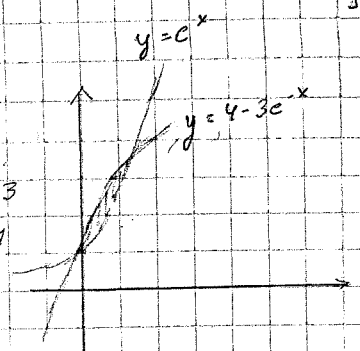
x	-	+	-	+	+	+
$2x^2 - 9$	+	+	-	+	+	+
d'(x)	-	+	-	+	+	+
d(x)	\	/	\	/	/	/

pieni  $d(\pm \frac{3}{\sqrt{2}}) = \sqrt{\frac{9}{2} + (5 - \frac{9}{2})^2} = 5$

$y = 5 - (\pm \frac{3}{\sqrt{2}})^2 = 5 - \frac{9}{2} = \frac{1}{2}$

Pisteet  $(\pm \frac{3}{\sqrt{2}}, \frac{1}{2})$

10.  $\begin{cases} y = e^x \\ y = 4 - 3e^{-x} \end{cases}$   
 $e^x = 4 - 3e^{-x} \mid \cdot e^x$   
 $e^{2x} = 4e^x - 3$   
 $e^{2x} - 4e^x + 3 = 0$   
 $e^x = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{4 \pm 2}{2} = 3$   
 $x = \ln 3 \vee x = \ln 1 = 0$



$A = \int_0^{\ln 3} (4 - 3e^{-x} - e^x) dx =$   
 $= [4x + 3e^{-x} - e^x]_0^{\ln 3} = 4 \ln 3 + \frac{3}{e^{\ln 3}} - e^{\ln 3} - (4 \cdot 0 + 3 \cdot e^0 - e^0)$   
 $= 4 \ln 3 + 1 - 3 - 3 + 1 = 4 \ln 3 - 4 \approx 0,3944..$   
V:  $4 \ln 3 - 4 \approx 0,394$

11. Pisteket osat  $\frac{a}{3}$   
 $2 \cdot \frac{1}{3} \cdot \frac{a}{3} = \frac{2}{9} a$   
 $2 \cdot \frac{1}{3} \cdot \frac{2}{9} a = \frac{4}{27} a$   
 yht  $\frac{a}{3} + \frac{2}{9} \cdot \frac{a}{3} + (\frac{2}{9})^2 a + \dots$  geom. summa,  $q = \frac{2}{9}$   
 $|q| < 1 \Rightarrow \text{supp.}$   
 $S = \frac{\frac{a}{3}}{1 - \frac{2}{9}} = a \Rightarrow \frac{S}{a} = 1$

12.  $m, n \in \mathbb{N}$   
 a)  $x_1 = 2m + 1, x_2 = 2n + 1$   
 $x_1 + x_2 = 2m + 1 + 2n + 1 = 2m + 2n + 2 = 2(m + n + 1) = 2p$   
 missä  $p \in \mathbb{N} \Rightarrow x_1 + x_2$  on parill.  $\square$

b)  $x_1 \cdot x_2 = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1$   
 $= 2(2mn + m + n) + 1 = 2q + 1$ , missä  $q \in \mathbb{N}$   
 $\Rightarrow x_1 \cdot x_2$  on pariton  $\square$

13.  $f: [0, 5] \rightarrow \mathbb{R}$  deriva,  $f(\frac{1}{2}) = 1$  ja

$\frac{1}{4} \leq f'(x) \leq \frac{3}{4}$

Väljarvolaus:  $\exists \xi \in [\frac{1}{2}, 3] : f(3) - f(\frac{1}{2}) = f'(\xi)(3 - \frac{1}{2})$   
 $f(3) = f'(\xi) \cdot \frac{5}{2} + f(\frac{1}{2}) = \frac{5}{2} \cdot f'(\xi) + 1$  ja  $\frac{1}{4} \leq f'(\xi) \leq \frac{3}{4}$

$\Rightarrow \frac{5}{2} \cdot \frac{1}{4} + 1 \leq f(3) \leq \frac{5}{2} \cdot \frac{3}{4} + 1$

$\Leftrightarrow \frac{13}{8} \leq f(3) \leq \frac{23}{8}$

14.  $p(x) = ax^3 + bx^2 + cx + d$ ,  $a \neq 0$

a)  $\int_0^2 p(x) dx = \int_0^2 (\frac{1}{4}ax^4 + \frac{1}{3}bx^3 + \frac{1}{2}cx^2 + dx)$   
 $= 4a + \frac{8}{3}b + 2c + 2d$

$\frac{1}{3}[p(0) + 4p(1) + p(2)] = \frac{1}{3}[d + 4a + 4b + 4c + 4d + 8a + 4b + 2c + d]$   
 $= \frac{1}{3}[12a + 8b + 6c + 6d] = 4a + \frac{8}{3}b + 2c + 2d = \int_0^2 p(x) dx$

b)  $\int_0^2 (x^3 + x^2 + x + 1) dx = \frac{1}{3}(1 + 4(1+1+1) + (8+4+2+1))$   
 $= \frac{1}{3} \cdot 32 = \frac{32}{3}$

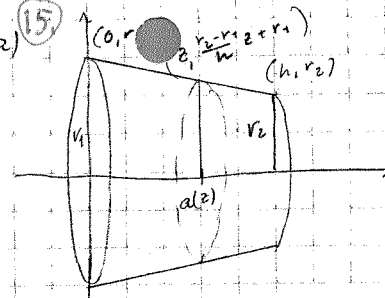
c) Esim.  $p(x) = x^4$

$\int_0^2 p(x) dx = \int_0^2 \frac{1}{5}x^5 = \frac{32}{5}$

$\frac{1}{3}[p(0) + 4p(1) + p(2)] = \frac{1}{3}[0 + 4 \cdot 1^4 + 2^4] = \frac{20}{3} \neq \frac{32}{5}$   
 $\Rightarrow$  laava ei päde esim. kun  $p(x) = x^4$

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a) 15



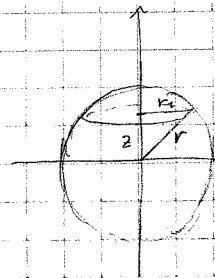
$k = \frac{r_2 - r_1}{h}$   
 $y = \frac{r_2 - r_1}{h}x + r_1$

$a(z) = \pi \left( \frac{r_2 - r_1}{h}z + r_1 \right)^2$

b)  $\int_0^h a(z) dz = \pi \int_0^h \left( \frac{r_2 - r_1}{h}z + r_1 \right)^2 dz =$   
 $= \pi \int_0^h \left( \frac{(r_2 - r_1)^2}{h^2}z^2 + \frac{2r_1(r_2 - r_1)}{h}z + r_1^2 \right) dz =$   
 $= \pi \left[ \frac{(r_2 - r_1)^2}{h^2} \cdot \frac{1}{3}z^3 + \frac{r_1(r_2 - r_1)}{h}z^2 + r_1^2z \right]_0^h =$   
 $= \pi \left( \frac{(r_2 - r_1)^2}{h^2} \cdot \frac{h^3}{3} + \frac{r_1(r_2 - r_1)}{h} \cdot h^2 + r_1^2h \right) =$   
 $= \frac{\pi h}{3} (r_1^2 + r_1r_2 + r_2^2)$

c) Kalk. karkon tilavuus

d)



$r_1 = \sqrt{r^2 - z^2}$

$V = \pi \int_{-r}^r \sqrt{r^2 - z^2} dz$

$= \pi \int_{-r}^r (r^2 - z^2) dz$

$= \pi \left[ r^2z - \frac{1}{3}z^3 \right]_{-r}^r = \pi \left[ r^3 - \frac{r^3}{3} - \left( -r^3 + \frac{r^3}{3} \right) \right]$

$= \frac{4}{3} \pi r^3$

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