

1. a) $(-x^2+2x)+(2x^2-3x+1) = -x^2+2x^2+2x-3x+1$
 $= x^2-x+1$

$(-x^2+2x)(2x^2-3x+1) = -2x^4+3x^3-x^2+4x^3-6x^2+2x$
 $= -2x^4+7x^3-5x^2+2x$

b) $\frac{x}{2} - \frac{x}{3} = x-1$
 $\frac{3x}{6} - \frac{2x}{6} = x-1 \quad | \cdot 6$
 $3x-2x = 6x-6$
 $-5x = -6 \quad | :5$
 $x = 1\frac{1}{5}$

c) $\begin{cases} x+y=1 \\ x-y=2 \end{cases}$
 $2x = 3 \Rightarrow x = 1\frac{1}{2}$
 $1\frac{1}{2} + y = 1$
 $y = -\frac{1}{2}$
 $V: \begin{cases} x = 1\frac{1}{2} \\ y = -\frac{1}{2} \end{cases}$

2a) $\frac{x}{2} = \frac{y}{5}$
 $\frac{2}{7} = \frac{5}{y} \Leftrightarrow 2y = 35$
 $y = 17\frac{1}{2}$

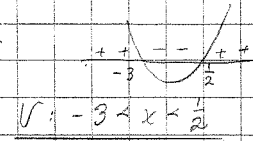
b) $f(x) = x^3 - 6x^2 + 1$
 $f'(x) = 3x^2 - 12x = 0$
 $x(3x-12) = 0$
 $x = 0 \vee 3x-12 = 0$
 $3x = 12$
 $x = 4$
 $V: x=0 \vee x=4$

c) $\frac{2x-1}{x+1} = \frac{5}{8}$
 $2 \cdot \frac{5}{8} - \frac{1}{1} = \frac{10}{8} - \frac{8}{8} = \frac{2}{8} = \frac{1}{4}$
 $\frac{5}{8} + \frac{1}{4} = \frac{5}{8} + \frac{2}{8} = \frac{7}{8}$
 $\frac{1}{4} = \frac{2}{8} = \frac{2 \cdot 8}{8 \cdot 13} = \frac{2}{13}$

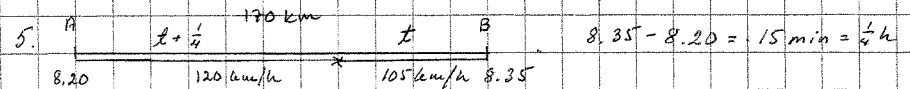
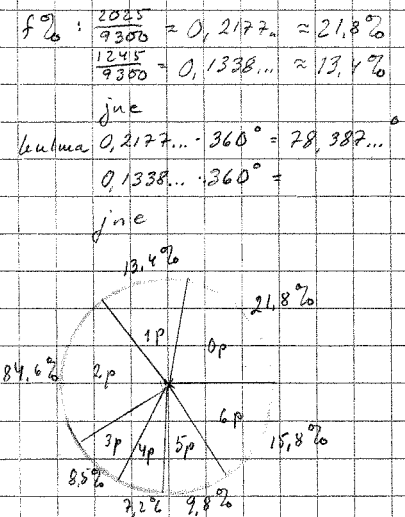
3. a) $k = -\frac{1}{3} \quad (-1, 2)$
 $y-2 = -\frac{1}{3}(x+1)$
 $y = -\frac{1}{3}x - \frac{1}{3} + 2$
 $y = -\frac{1}{3}x + 1\frac{2}{3}$

b) $2x^2+5x-3 < 0$
 nolake. $2x^2+5x-3 = 0$
 $x = \frac{-5 \pm \sqrt{25-4 \cdot 2 \cdot (-3)}}{2 \cdot 2} = \frac{-5 \pm 7}{4}$
 $x = \frac{1}{2} \vee x = -3$

merujuk:



pangkat	f	f%	kuilua
0	2025	21,8	78,4°
1	1245	13,4	48,2°
2	2186	23,5	84,6°
3	792	8,5	30,7°
4	673	7,2	26,1°
5	908	9,8	35,1°
6	1471	15,8	56,9°
n = 9360			



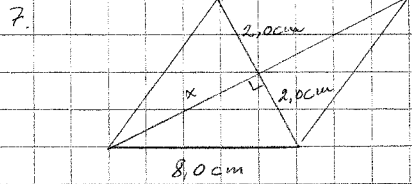
$8:35 - 8:20 = 15 \text{ min} = \frac{1}{4} \text{ h}$
 $s = vt$
 $120 \cdot (t + \frac{1}{4}) + 105 \cdot t = 170$
 $120t + 30 + 105t = 170$
 $225t = 140 \quad | :225$
 $t = 0,6222... \text{ h} = 37 \text{ min } 20 \text{ s (laskimesk)}$

$120 \cdot (0,6222... + \frac{1}{4}) = 104,666...$
 $8:35 + 0,37 \cdot 20 = 9:12:20$
 $V: 104,7 \text{ km piasissa A:st klo } 9:12$

6. $2\pi r = 64,2 / (2\pi)$ $m = 37,9 \text{ kg}$
 $r = \frac{32,1}{\pi}$ $\rho = 8,96 \text{ g/cm}^3$

$V = \frac{4}{3} \pi \cdot \left(\frac{32,1}{\pi}\right)^3 = 4468,42... \text{ (cm}^3\text{)}$
 $m_p = \rho V = 8,96 \cdot 4468,42... \text{ g} = 40037,0... \text{ g} \approx 40,0 \text{ kg}$

Umpipallo painaisi yli 2kg enemmän kuin pallo painaa, joten sisällä on tyhjää



$x^2 + 2^2 = 8^2$
 $x^2 = 64 - 4$
 $x = \pm \sqrt{60} = 7,745... \text{ cm}$
 $2x = 15,49 \text{ cm}$

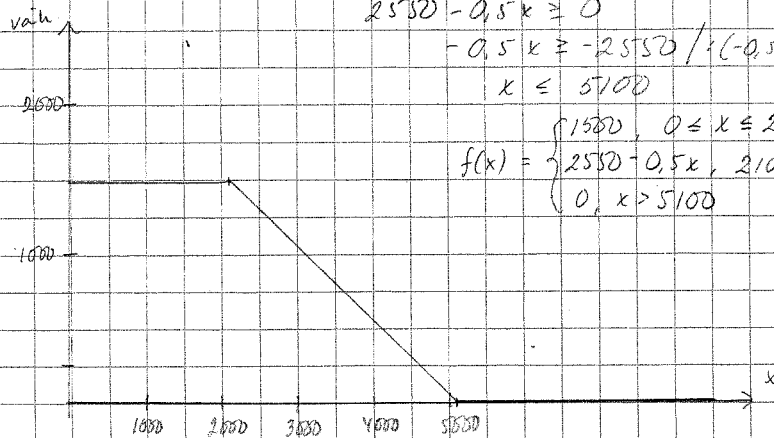
$V: 15 \text{ cm}$

8. palkkatulo x
 ansiotulo vähennys kun $x + 1500 > 2200 \Leftrightarrow x > 700$
 $x + 1500$ $2200 - 0,5(x + 1500 - 2200)$
 $= 2200 - 0,5x - 750 + 1100 = 2550 - 0,5x$

Korko: $1500: 2550 - 0,5x \leq 1500$
 $-0,5x \leq -1050 / (-0,5)$
 $x \geq 2100$

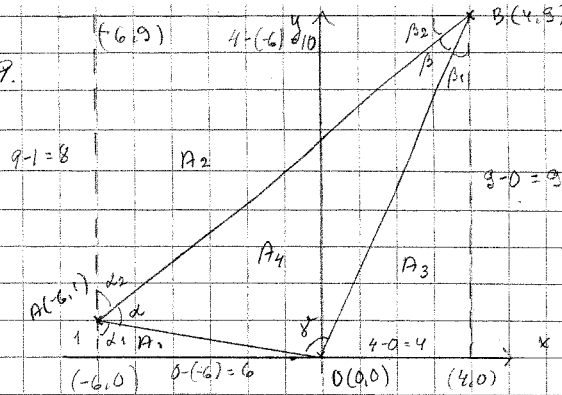
$2550 - 0,5x \geq 0$
 $-0,5x \geq -2550 / (-0,5)$
 $x \leq 5100$

$f(x) = \begin{cases} 1500, & 0 \leq x \leq 2100 \\ 2550 - 0,5x, & 2100 < x \leq 5100 \\ 0, & x > 5100 \end{cases}$



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9.

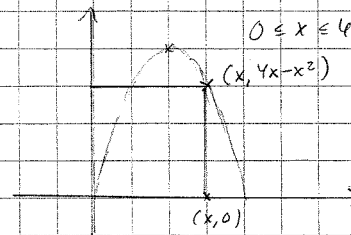


$\tan \alpha_1 = \frac{6}{1} \Rightarrow \alpha_1 = 80,537...^\circ$ $\alpha = 180^\circ - \alpha_1 - \alpha_2 = 48,122...^\circ$
 $\tan \alpha_2 = \frac{10}{8} \Rightarrow \alpha_2 = 51,340...^\circ$
 $\tan \beta_1 = \frac{4}{9} \Rightarrow \beta_1 = 23,962...^\circ$ $\beta = 90^\circ - \beta_1 - \beta_2 = 27,377...^\circ$
 $\tan \beta_2 = \frac{8}{10} \Rightarrow \beta_2 = 38,659...^\circ$ $\gamma = 180^\circ - \alpha - \beta = 104,500...^\circ$

a) $V: \alpha = 48,1^\circ, \beta = 27,4^\circ, \gamma = 104,5^\circ$

b) $A = 9 \cdot 10 = 90$ $A_4 = 90 - 3 - 40 - 18 = 29$
 $A_1 = \frac{6 \cdot 1}{2} = 3$
 $A_2 = \frac{8 \cdot 10}{2} = 40$
 $A_3 = \frac{4 \cdot 9}{2} = 18$

10.



$y = 4x - x^2$
 $y' = 4 - 2x = 0$
 kumpin $x = 2$
 $y = 4 \cdot 2 - 2^2 = 4$

$4x - x^2 = 0$
 $x(4 - x) = 0$
 $x = 0 \vee x = 4$

$A(x) = x(4x - x^2) = 4x^2 - x^3$
 $A'(x) = 8x - 3x^2 = 0$
 $x(8 - 3x) = 0$
 $x = 0 \vee 8 - 3x = 0$
 $x = \frac{8}{3} = 2\frac{2}{3}$

$A''(x) = 8 - 6x$
 $A''(1) = 8 - 3 = 5$
 $A''(3) = 8 - 3 \cdot 3 = -3$

Suurin ala $A\left(\frac{8}{3}\right) = \frac{8}{3} \left(4 \cdot \frac{8}{3} - \left(\frac{8}{3}\right)^2\right) = \frac{8}{3} \left(\frac{32}{3} - \frac{64}{9}\right) = \frac{8}{3} \cdot \frac{32}{9} = \frac{256}{27}$

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11. $P(V) = 10\%$ $P(\bar{V}) = 90\%$

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$$P(\text{ain. } V) = 1 - P(\text{ei yhtään } V) = 1 - 0,9^n = 0,8$$

$$-0,9^n = -0,2 \quad | \cdot (-1)$$

$$0,9^n = 0,2 \quad | \lg$$

$$n \lg 0,9 = \lg 0,2 \quad | : \lg 0,9$$

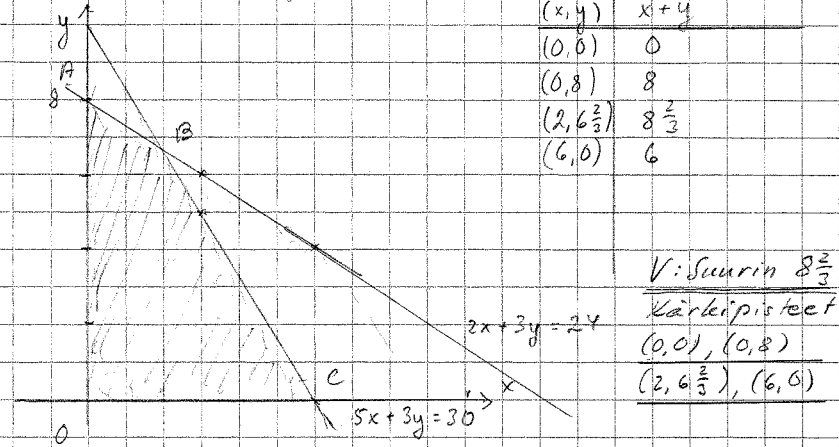
$$n = \frac{\lg 0,2}{\lg 0,9} = 15,27\dots$$

V: väk. 16

12. $\begin{cases} x \geq 0 \\ y \geq 0 \\ 2x + 3y \leq 24 \\ 5x + 3y \leq 30 \end{cases}$

$$2x + 3y \leq 24 \Leftrightarrow 3y \leq 24 - 2x \Leftrightarrow y \leq 8 - \frac{2}{3}x$$

$$5x + 3y \leq 30 \Leftrightarrow 3y \leq 30 - 5x \Leftrightarrow y \leq 10 - \frac{5}{3}x$$



V: suurin $8\frac{2}{3}$
Kärkipisteet
(0,0), (0,8)
(2, $6\frac{2}{3}$), (6,0)

$O = (0,0)$

$A: \begin{cases} x = 0 \\ y = 8 - \frac{2}{3}x \end{cases} \Rightarrow A = (0,8)$

$B: \begin{cases} 2x + 3y = 24 \\ 5x + 3y = 30 \end{cases} \cdot (-1) \Rightarrow \begin{cases} -2x - 3y = -24 \\ 5x + 3y = 30 \end{cases}$

$$3x = 6$$

$$x = 2$$

$B = (2, 6\frac{2}{3})$

$C: \begin{cases} y = 0 \\ 5x + 3y = 30 \end{cases} \Rightarrow 5x = 30 \quad : 5$

$$x = 6$$

$$3y = 20 \quad | : 3$$

$$y = 6\frac{2}{3}$$

$C = (6,0)$

13. $a_1 = 1$

$a_n = 61$

$S_n = n \cdot \frac{1+61}{2} = 961$

$n \cdot 31 = 961 \quad | : 31$

$n = 31$

$a_{21} = 1 + 30d = 61$

$30d = 60$

$d = 2$

$a_2 = a_1 + d = 1 + 2 = 3$

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14. tod. korko $(1 - 0,29) \cdot 1,50\% = 0,71 \cdot 1,50\% = 1,065\%$

a) $1000 \cdot 1,01065^{10} = 1111,751\dots$ V: 1111,75 €

b) $1000 \cdot 1,01065^x = 2000 \quad | : 1000$

$$1,01065^x = 2 \quad | \lg$$

$$x \lg 1,01065 = \lg 2 \quad | : \lg 1,01065$$

$$x = 65,43\dots$$

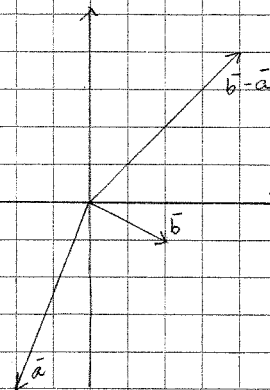
V: 66 vuodet

15. $\vec{a} = -2\vec{i} - 5\vec{j}$

$\vec{b} = 2\vec{i} - \vec{j}$

a) $\vec{b} - \vec{a} = 2\vec{i} - \vec{j} - (-2\vec{i} - 5\vec{j}) = 2\vec{i} - \vec{j} + 2\vec{i} + 5\vec{j} = 4\vec{i} + 4\vec{j}$

b)



c) $\vec{a} \cdot \vec{b} = -2 \cdot 2 - 5 \cdot (-1) = 1$

$|\vec{a}| = \sqrt{2^2 + 5^2} = \sqrt{29}$

$|\vec{b}| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\cos(\vec{a}, \vec{b}) = \frac{1}{\sqrt{29} \cdot \sqrt{5}}$

$\angle(\vec{a}, \vec{b}) = 85,236\dots^\circ \approx 85,2^\circ$