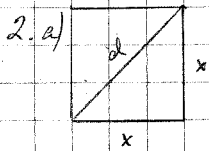


1. a) $\frac{1}{2}(3x-2) = \frac{1}{3}(2x+3)$
 $\frac{3}{2}x - 1 = \frac{2}{3}x + 1$
 $x - \frac{2}{3}x = 1 + 1$
 $\frac{1}{3}x = 2 \quad | \cdot \frac{3}{1}$
 $x = 2 \frac{2}{3}$

b) $(x+2)(x-2) = 5$
 $x^2 - 2x + 2x - 4 = 5$
 $x^2 = 9 \quad | \sqrt{\quad}$
 $x = \pm 3$

c) $\begin{cases} x+y=2 \\ 2x-y=5 \end{cases}$
 $3x = 7$
 $x = \frac{7}{3} = 2 \frac{1}{3}$
 $2 \frac{1}{3} + y = 2$
 $y = 2 - 2 \frac{1}{3} = -\frac{1}{3}$

$V: (2 \frac{1}{3}, -\frac{1}{3})$



$d^2 = x^2 + x^2 = 2x^2 = 2 \cdot 1,20 = 2,40$
 $d^2 = 2,40 \quad | \sqrt{\quad}$
 $d = 1,549...$

$A = x^2 = 1,20 \text{ m}^2$

$V: d = 1,55 \text{ m}$

b) $x^4 = 17 \quad | \sqrt[4]{\quad}$
 $x = \sqrt[4]{17} = 2,0305... \approx 2,031$

c) $4^0 - 3^{-1} + 2^{-2} - 1^{-3} = 1 - \frac{1}{3} + \frac{1}{2^2} - \frac{1}{1^3} = 1 - \frac{1}{3} + \frac{1}{4} - 1$
 $= -\frac{1}{12}$

3 a) f 'in 0-kohdat x -akselilla
 $x = -2$ v $x = 0,5$ v $x = 2,8$

b) f 'in 0-kohdat = käännyksi kohdat
 $x = -1$ v $x = 2$

c) suurin arvo $f(3) = 5$

d) pienin arvo $f(2) = -5$

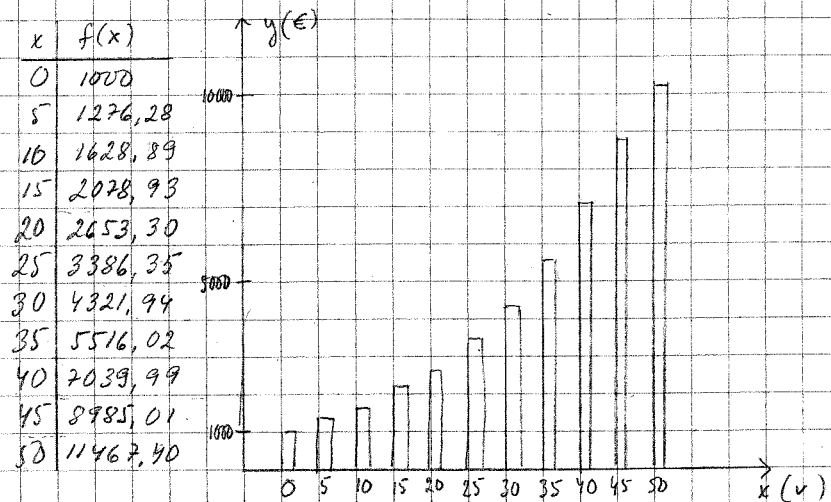
e) $-2 \leq x \leq -1$ tai $2 \leq x \leq 3 \Rightarrow f$ kasvava

f) $-1 < x < 0$ f vähenevä

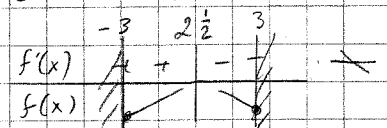
4. suolaa: $0,12x + 0,05 \cdot 3 = 0,08(x+3)$
 $0,12x + 0,15 = 0,08x + 0,24$
 $0,04x = 0,09 \quad | : 0,04$
 $x = 2,25 \quad V: 2,25 \text{ l}$

5. 1000e, 5% 50 v

$f(x) = 1000 \cdot 1,05^x$



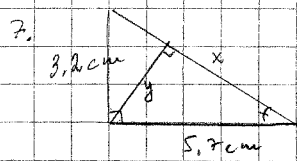
6. $f(x) = (x-2)(3-x) = 3x - x^2 - 6 + 2x = -x^2 + 5x - 6$
 $f'(x) = -2x + 5 = 0$
 $-2x = -5 \quad | : (-2)$
 $x = 2 \frac{1}{2}$



Suurin arvo $f(2 \frac{1}{2}) = (2 \frac{1}{2} - 2)(3 - 2 \frac{1}{2}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Pienin arvo jolloin $f(-3) = (-3-2)(3+3) = -5 \cdot 6 = -30$
 tai $f(3) = (3-2)(3-3) = 1 \cdot 0 = 0$

Pienin -30, suurin $\frac{1}{4}$



$$x^2 = 3,2^2 + 5,7^2$$

$$x^2 = 42,73 \quad | \sqrt{\quad}$$

$$x = 6,536...$$

alkup. ja oikeanpuol. s ~ (KK)

$$\frac{y}{3,2} = \frac{5,7}{x}$$

$$xy = 5,7 \cdot 3,2 \quad | :x$$

$$y = \frac{18,24}{6,536...} = 2,790...$$

V: Hypot. 6,5 cm ja etäisyys 2,8 cm

8. $P(\text{virhe}) = P(V) = 0,00015$ $P(\bar{V}) = 0,99985$
 $P(\text{jonossa kaikki oikein}) = 0,99985^{16}$

a) $P(\text{ain. 1 virhe}) = 1 - P(\text{ei virheitä})$
 $= 1 - 0,99985^{16} = 0,002397... \approx \underline{0,0024}$

b) $P(\text{ain. 1 virheell. jono}) = 1 - P(\text{ei virheell. jonoja})$
 $= 1 - (0,99985^{16})^{32} = 0,0739... \approx \underline{0,074}$

9. $A = (-1, 1)$ $B = (8, 4)$

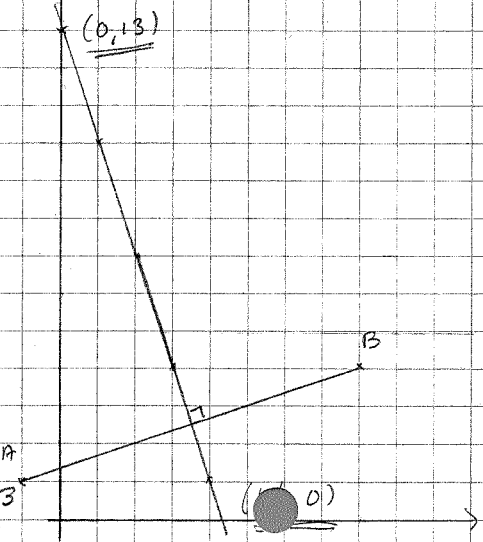
keskipisteen $x = \frac{-1+8}{2} = \frac{7}{2}$
 $y = \frac{1+4}{2} = \frac{5}{2}$

$k_{AB} = \frac{4-1}{8-(-1)} = \frac{3}{9} = \frac{1}{3}$
 $\frac{1}{3} \cdot k = -1 \Rightarrow k = -3$

$y - \frac{5}{2} = -3(x - \frac{7}{2})$
 $y = -3x + \frac{21}{2} + \frac{5}{2}$
 $y = -3x + 13$

x-akseli $y = 0$
 $-3x + 13 = 0$
 $-3x = -13$
 $x = \frac{13}{3} = 4\frac{1}{3}$

y-akseli $x = 0$
 $y = -3 \cdot 0 + 13 = 13$



MA B
K10

10.



$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \cdot 3^2 \cdot 2 = \underline{6\pi \approx 18,8}$$

MA B
K10

11. $a_{n+1} = \frac{1}{2} (a_n + \frac{2}{a_n})$ $\frac{1}{2} (ANS + \frac{2}{ANS})$

a) $a_0 = 3$
 $a_1 = \frac{1}{2} (3 + \frac{2}{3}) = \frac{11}{6} = 1,833... \approx 1,833333333$
 $a_2 = 1,46212... \approx 1,462121212$
 $a_3 = 1,4149... \approx 1,414998430$
 $a_4 = 1,4142... \approx 1,414213780$ $\frac{a_4}{\sqrt{2}} = 1,000000154...$
On n. 0,000015 % suurempi

b) $a_0 = 8$
 $a_1 = \frac{1}{2} (8 + \frac{2}{8}) = 4,125$
 $a_2 = 2,3049... \approx 2,304924242$ $\frac{a_4}{\sqrt{2}} = 1,00660...$
 $a_3 = 1,5863... \approx 1,586315860$
 $a_4 = 1,4235... \approx 1,423549408$
On n. 0,66 % suurempi

12.

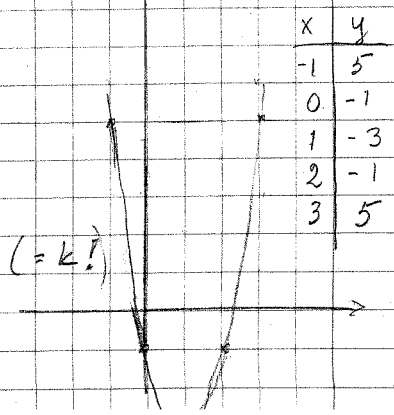
$y = 2x^2 - 4x - 1$
 $y - k = a(x - h)^2$
 $y - k = a(x^2 - 2xh + h^2)$
 $y = ax^2 - 2ahx + ah^2 + k = 2x^2 - 4x - 1$
 $\begin{cases} a = 2 \\ -2ah = -4 \end{cases} \Rightarrow -4h = -4 \Rightarrow h = 1$
 $ah^2 + k = -1 \quad 2 \cdot 1^2 + k = -1 \Rightarrow k = -3$

$a = 2, h = 1, k = -3$

$y = 2x^2 - 4x - 1$
 $y' = 4x - 4 = 0$
 $4x = 4$

huipun $x = 1$ (=h!)
 $y = 2 \cdot 1^2 - 4 \cdot 1 - 1 = -3$ (=k!)

Huippu $(1, -3)$

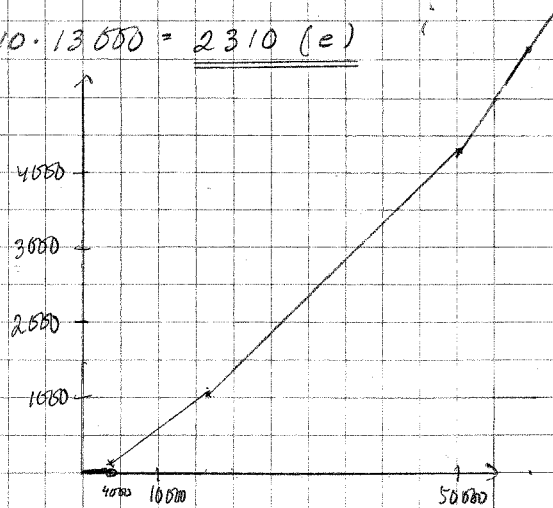


13. a) $30000 - 17000 = 13000$

MAB
K10

Vero $1010 + 0,10 \cdot 13000 = 2310$ (e)

| x | y |
|--------|--------------------------------------|
| 40000 | 100 |
| 170000 | 1010 |
| 500000 | 4310 |
| 600000 | $4310 + 0,13 \cdot 100000 =$ 5610 |

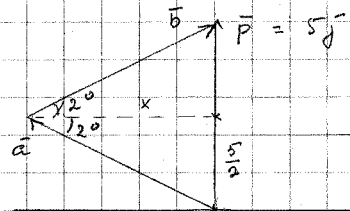


14. $2083 \quad 194,26 \text{ €} \quad 2009 \quad 249 \text{ €}$

$194,26 \cdot x^6 = 249 \quad | : 194,26$
 $x^6 = 1,2817... \quad | \sqrt[6]{\quad}$
 $x = 1,0422...$

V: 4,2%

15



$\vec{a} = -x\vec{i} + y\vec{j}$
 $\vec{b} = x\vec{i} + y\vec{j}$
 $\vec{p} = \vec{a} + \vec{b}$
 $5\vec{j} = -x\vec{i} + y\vec{j} + x\vec{i} + y\vec{j}$
 $5\vec{j} = 2y\vec{j}$
 $2y = 5$
 $y = \frac{5}{2}$

$\tan 2^\circ = \frac{\frac{5}{2}}{x}$
 $x \tan 2^\circ = \frac{5}{2}$
 $x = \frac{5}{2 \tan 2^\circ} = 71,590...$

$x \approx 71,6 \quad y = 2,5$