

$$1a) \quad 4x + (5x - 4) = 12 + 3x$$

$$4x + 5x - 4 = 12 + 3x$$

$$6x = 16 \quad | :6$$

$$x = \underline{\underline{2\frac{2}{3}}}$$

$$b) \quad x^2 + x - (x^2 - x) = x^2 + x - x^2 + x = \underline{\underline{2x}}$$

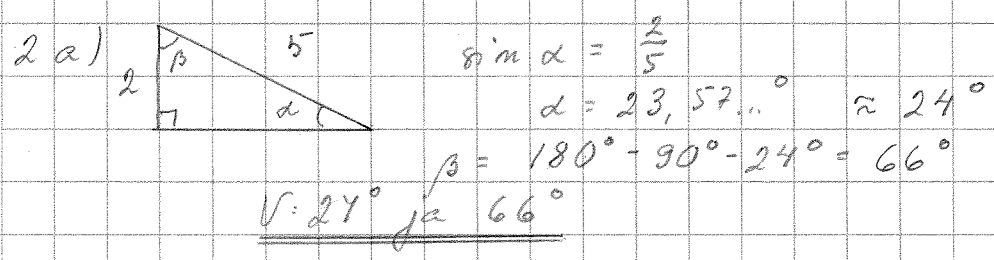
$$x = \frac{1}{2} \Rightarrow 2x = 2 \cdot \frac{1}{2} = \underline{\underline{1}}$$

$$c) \quad \begin{cases} x - 2y = 0 \\ x - 3y = 1 \end{cases} \quad | \cdot (-1) \quad \begin{matrix} x - 2 \cdot (-1) = 0 \\ x = -2 \\ y = \underline{\underline{-1}} \end{matrix}$$

$$\begin{cases} -x + 2y = 0 \\ x - 3y = 1 \end{cases}$$

$$\underline{\quad \quad \quad}$$

$$-y = 1 \Rightarrow y = -1$$



$$b) \quad (\sqrt{x} - 1)^2 + 2\sqrt{x} = (\sqrt{x} - 1)(\sqrt{x} - 1) + 2\sqrt{x}$$

$$= \sqrt{x}^2 - \sqrt{x} - \sqrt{x} + 1 + 2\sqrt{x}$$

$$= x - 2\sqrt{x} + 1 + 2\sqrt{x} = \underline{\underline{x + 1}}$$

$$c) \quad |x - y| = |2 - 5| = |-3| = \underline{\underline{3}}$$

$$3a) \quad x^2 - 4ax + 4a^2 = 0 \quad | x = 2$$

$$4 - 4a \cdot 2 + 4a^2 = 0$$

$$4a^2 - 8a + 4 = 0$$

$$a = \frac{8 \pm \sqrt{64 - 4 \cdot 4 \cdot 4}}{2 \cdot 4} = \frac{8 \pm 0}{8} \quad \underline{\underline{a = 1}}$$

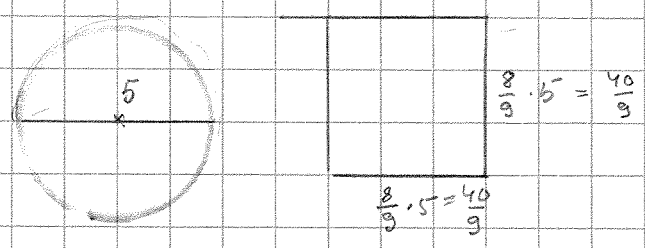
$$b) \quad a \cdot 1,20 \cdot 0,83 = 0,996 a$$

$$0,996 = 99,6 \%$$

$$(100 - 99,6) \% = \underline{\underline{0,4 \% \text{ pienempi}}}$$

(kasvaa 20% \rightarrow 120% = 1,2, vähenee 17%
 (100 - 17) % = 83% = 0,83)

4.



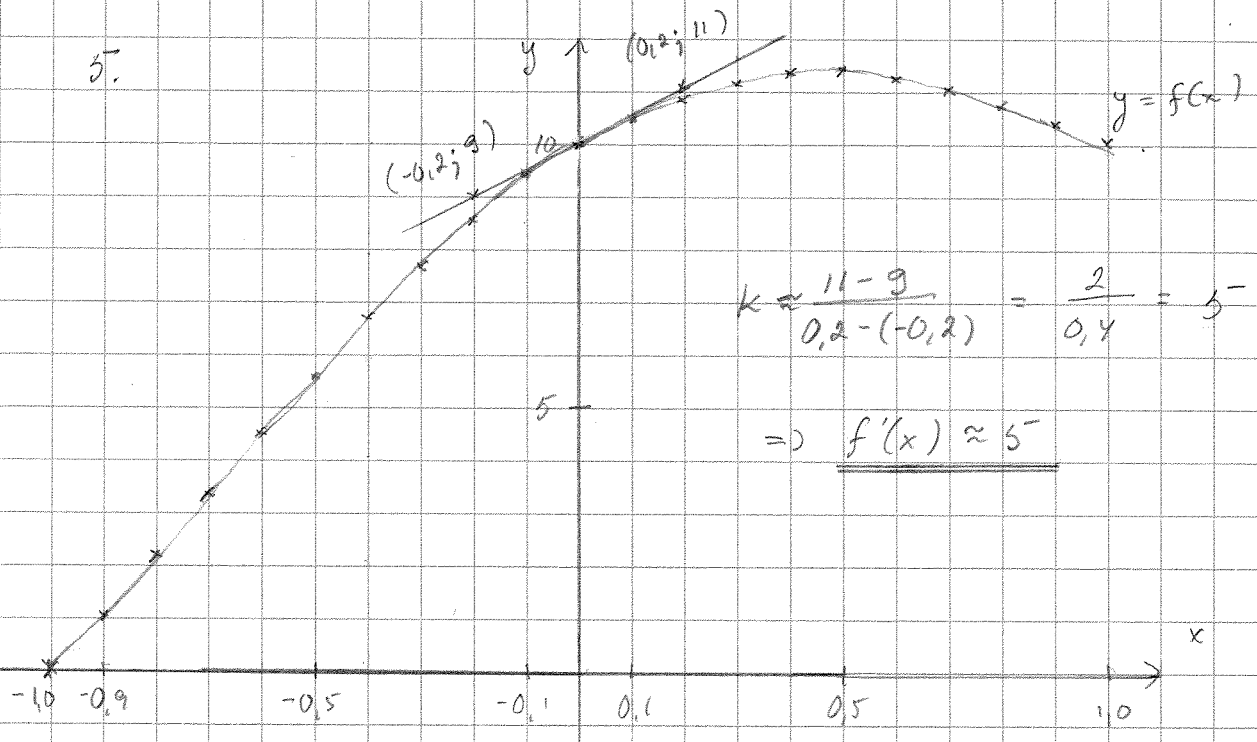
a) $A \approx \left(\frac{40}{3}\right)^2 = 19 \frac{61}{81} = 19,753... \approx 19,8$

b) $A = \pi r^2 = \pi \cdot \left(\frac{5}{2}\right)^2 = \frac{25}{4} \pi = 19,634...$

$\frac{19 \frac{61}{81}}{\frac{25}{4} \pi} = 1,00601...$
 $(100,6 - 100)\% = 0,6\%$

V: Saatu ala on 0,6%
liian suuri

5.



$k = \frac{11-9}{0,2-(-0,2)} = \frac{2}{0,4} = 5$
 $\Rightarrow \underline{f'(x) \approx 5}$

6.

$k = 1 : 20000$
 $\frac{A_5}{A_4} = \frac{1}{2} \approx \left(\frac{x}{1}\right)^2$

A4: Ua 1 cm, A5: Ua x cm
 luonnossa kolo ajan 20000 cu

$\frac{1}{2} = \frac{x^2}{1}$
 $2x^2 = 1 \quad | :2$
 $x^2 = \frac{1}{2}$

$x = \pm \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

mittakaava on

$\frac{x}{20000} = \frac{1}{20000\sqrt{2}}$
 $= \frac{1}{28284,27...}$
 $\approx \frac{1}{28000}$

7.

$$\bar{x} = \frac{5,80 \cdot 0 + 10,99 \cdot 1 + 17,54 \cdot 2 + 24,78 \cdot 3 + 19,95 \cdot 4 + 15,48 \cdot 5 + 5,46 \cdot 6}{100}$$

$$= \frac{310,37}{100} = 3,1037 \approx \underline{\underline{3,1}}$$

Laskimesta

$$s_n = 1,57608... \approx \underline{\underline{1,56}}$$

x	f%	$x - \bar{x}$	$(x - \bar{x})^2$	$f\% \cdot (x - \bar{x})^2$
0	5,80	-3,1037		
1	10,99	-2,1037		
2	17,54	-1,1037		
3	24,78	-0,1037		
4	19,95	0,8963		
5	15,48	1,8963		
6	5,46	2,8963		

$$s = \sqrt{\frac{\sum p_i \cdot (x_i - \bar{x})^2}{\sum p_i}}$$

8. $P(R) = \frac{1}{2}$ $P(S) = \frac{1}{2}$

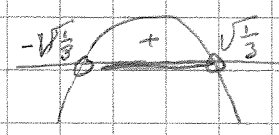
a) $P(\text{kuuio R}) = P(RSRSRSR) = \left(\frac{1}{2}\right)^3 = 0,00195... \approx \underline{\underline{0,20\%}}$

b) $P(\text{rivi ei yksivärinen}) = 1 - P(3S \text{ tai } 3R)$
 $= 1 - \left(\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3\right) = \frac{3}{4} = \underline{\underline{75\%}}$

$P(\text{mikään rivi ei yksivärinen}) = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$
 $= 0,4218... \approx \underline{\underline{42\%}}$

9. $f(x) = -x^3 + x + 2$
 $f'(x) = -3x^2 + 1$
 $f'(x) > g'(x)$
 $-3x^2 + 1 > 3x^2 - 1$
 $-6x^2 + 2 > 0$

$g(x) = x^3 - x - 2$
 $g'(x) = 3x^2 - 1$

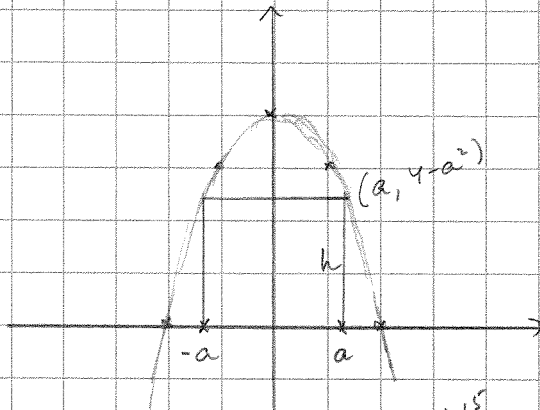


0-kohtat $-6x^2 + 2 = 0$
 $-6x^2 = -2 \quad | :(-6)$
 $x^2 = \frac{1}{3}$
 $x = \pm \sqrt{\frac{1}{3}}$

$-\sqrt{\frac{1}{3}} < x < \sqrt{\frac{1}{3}}$

10.

$0 < a < 2$

K-11
MA13

$$h = 4 - a^2$$

$$\text{kanta } a - (-a) = 2a$$

$$A(a) = 2a(4 - a^2) = 8a - 2a^3$$

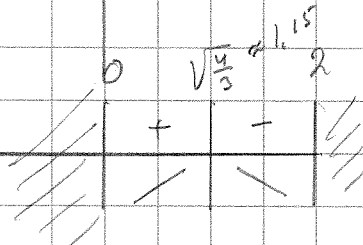
$$A'(a) = 8 - 6a^2$$

$$8 - 6a^2 = 0$$

$$-6a^2 = -8 \quad | :(-6)$$

$$a^2 = \frac{4}{3}$$

$$a = \pm \sqrt{\frac{4}{3}}$$

 $A'(a)$ $A(a)$ 

$$A'(1) = 8 - 6 \cdot 1^2 = 2$$

$$A'(1,5) = 8 - 6 \cdot 1,5^2 = -5,5$$

V: Ala suurin, kun

$$a = \sqrt{\frac{4}{3}} \left(= \frac{2}{\sqrt{3}} \right) \approx 1,15$$

11. x on muutos vrt:ssa

$$25,0 \cdot x^5 = 16,2 \quad | : 25,0$$

$$x^5 = 0,648 \quad | \sqrt[5]{}$$

$$x = 0,91688\dots$$

$$x \cdot 0,91688\dots^t = 0,5 \quad | \lg$$

$$t \lg 0,91688\dots = \lg 0,5 \quad | : \lg 0,91688\dots$$

$$t = \frac{\lg 0,5}{\lg 0,91688\dots} = 7,988\dots$$

V: 8,0 vrt

$$25,0 \cdot 0,91688\dots^{-10} = 59,53\dots$$

V: 59,5 kBq

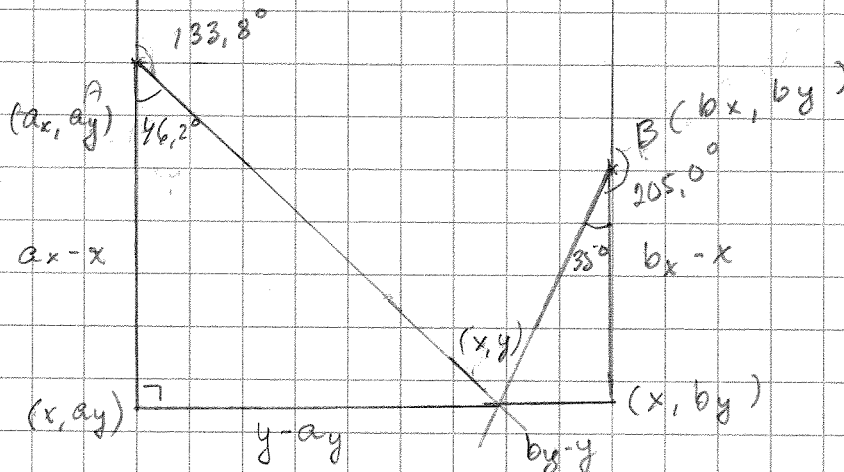
12.

$$180^\circ - 133,8^\circ = 46,2^\circ$$

$$205^\circ - 180^\circ = 25^\circ$$

K-11

MMB



$$A = (6670801, 2549572)$$

$$B = (6670015, 2554955)$$

$$\tan 46,2^\circ = \frac{y - a_y}{a_x - x}$$

$$\tan 25^\circ = \frac{b_y - y}{b_x - x}$$

$$(a_x - x) \tan 46,2^\circ = y - a_y$$

$$(b_x - x) \tan 25^\circ = b_y - y$$

$$y = (a_x - x) \tan 46,2^\circ + a_y$$

$$y = b_y - (b_x - x) \tan 25^\circ$$

Leikk. pist. $(a_x - x) \tan 46,2^\circ + a_y = b_y - (b_x - x) \tan 25^\circ$

$$a_x \tan 46,2^\circ - x \tan 46,2^\circ + a_y = b_y - b_x \tan 25^\circ + x \tan 25^\circ$$

$$x (\tan 46,2^\circ + \tan 25^\circ) = a_x \tan 46,2^\circ + a_y - b_y + b_x \tan 25^\circ$$

$$x = \frac{a_x \tan 46,2^\circ + a_y - b_y + b_x \tan 25^\circ}{\tan 46,2^\circ + \tan 25^\circ} = \frac{10061143,04}{1,50909...}$$

$$x \approx 6666991$$

$$y = (a_x - x) \tan 46,2^\circ + a_y \approx 2553545$$

$$V: (6666991, 2553545)$$

13. $a_1 = 10$, $a_2 = 12$ aritm., $d = 12 - 10 = 2$

$b_1 = 2$, $q = \frac{21}{20}$ geom

$$a_n = 10 + (n-1) \cdot 2 = 10 + 2n - 2 = 2n + 8$$

$$b_n = 2 \cdot \left(\frac{21}{20}\right)^{n-1}$$

$$2 \cdot \left(\frac{21}{20}\right)^{n-1} > 2n + 8$$

n	a_n	b_n
10	28	$> 3,102...$
50	108	$> 21,84...$
100	208	$< 250,17...$
90	188	$> 153,77...$
95	198	$> 196,25...$
96	200	$< 206,06...$

V: 96: nnessä lähtien

$$14. a) 0,054 \cdot 1,8 \cdot 10^6 = 97200$$

$$\text{pääomaan } 0,3 \cdot 97200 = 29160$$

$$97200 - 29160 - 2 \cdot 21000 = 14x$$

$$14x = 26040 \quad || :14$$

$$x = 1860$$

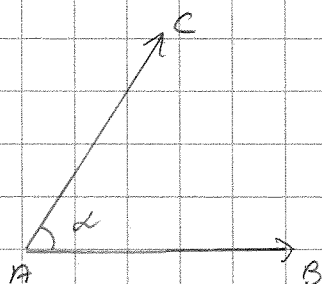
$$\underline{V: 1860 \text{ e}}$$

$$b) \text{ alku } 1,8 \cdot 10^6$$

$$\text{lisäys } 30\% \text{ tuotosta eli } 0,3 \cdot 0,054 = 0,0162$$

$$= 1,8 \cdot 10^6 \cdot 1,0162^5 = 1950601,0... \approx \underline{1,95 \text{ milj. e}}$$

15.



$$\angle BAC = \alpha$$

$$A = (1, 2, 3)$$

$$B = (4, 5, 6)$$

$$C = (9, 8, 7)$$

$$\overline{AB} = (4-1)\vec{i} + (5-2)\vec{j} + (6-3)\vec{k} = 3\vec{i} + 3\vec{j} + 3\vec{k}$$

$$\overline{AC} = (9-1)\vec{i} + (8-2)\vec{j} + (7-3)\vec{k} = 8\vec{i} + 6\vec{j} + 4\vec{k}$$

$$\overline{AB} \cdot \overline{AC} = 3 \cdot 8 + 3 \cdot 6 + 3 \cdot 4 = 54$$

$$|\overline{AB}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{27}$$

$$|\overline{AC}| = \sqrt{8^2 + 6^2 + 4^2} = \sqrt{116}$$

$$\cos \alpha = \frac{\overline{AB} \cdot \overline{AC}}{|\overline{AB}| \cdot |\overline{AC}|} = \frac{54}{\sqrt{27} \cdot \sqrt{116}}$$

$$\alpha = 15,22...^\circ \approx \underline{\underline{15^\circ}}$$