

1a) $7x + 3 = 31$
 $7x = 28 \quad | :7$
 $x = 4$

c) $\begin{cases} 2x - y = 1 \\ x + y = 8 \end{cases}$
 $3x = 9 \Rightarrow x = 3$
 $3 + y = 8 \Rightarrow y = 5$
V: $x = 3$ ja $y = 5$

b) $\frac{2a + 3b}{a - b}$ $a = \frac{5}{2}$ $b = \frac{7}{3}$
 $\frac{2 \cdot \frac{5}{2} + 3 \cdot \frac{7}{3}}{\frac{5}{2} - \frac{7}{3}} = \frac{5 + 7}{\frac{15}{6} - \frac{14}{6}} = \frac{12}{\frac{1}{6}} = 12 \cdot 6 = 72$

2.

f(x)	x^2	$\frac{1}{x}$	x	\sqrt{x}	x^3	x
kuva	2	4	1	6	5	3
	parabeli		suora		pariton pot.	

3. a) $\frac{7x + \frac{1}{2}}{3} - \frac{3x - \frac{1}{2}}{2} = 2$
 $\frac{14x + 1}{6} - \frac{9x - 1}{6} = \frac{12}{6} \quad | \cdot 6$
 $14x + 1 - (9x - 1) = 12$
 $14x + 1 - 9x + 1 = 12$
 $5x = 10$
 $x = 2$

b) $27^{x-2} = 9^{\frac{x}{2}}$ tai otetaan lg
 $(3^3)^{x-2} = (3^2)^{\frac{x}{2}}$ $(x-2) \lg 27 = \frac{x}{2} \lg 9 \quad | : \lg 27$
 $3^{3x-6} = 3^x$ $x - 2 = \frac{x}{2} \frac{\lg 9}{\lg 27}$
 $3x - 6 = x$ $x = \frac{\lg 9}{2 \lg 27} x + 2$
 $2x = 6$ $x = 0,333... x + 2$
 $x = 3$ $0,666... x = 2 \quad | : 0,666...$
 $x = 3$

4a) $f(x) = \frac{3}{2}x + b$ $f(x) = 0$, kun $x = 2 \Rightarrow f(2) = 0$
 $\frac{3}{2} \cdot 2 + b = 0$
 $b = -3$

b) $f(x) = \frac{3}{2}x - 3$ leikkaa y-akseli pist. $(0, -3)$
 $(f(0) = -3)$

c) $k = \frac{3}{2}$
 $\tan \alpha = \frac{3}{2} \Rightarrow \alpha = 56,309...^\circ$ V: $\alpha = 56,3^\circ$

5. $f(x) = (x+3)(x^2-4)$

a) $(x+3)(x^2-4) = 0$

$x+3=0 \vee x^2-4=0$

$x = -3 \vee x^2 = 4$

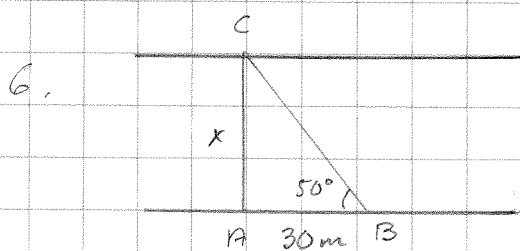
$x = -3 \vee x = \pm 2$

b) $f(x) = (x+3)(x^2-4) = x^3 + 3x^2 - 4x - 12$

$f'(x) = 3x^2 + 6x - 4$

c) $3x^2 + 6x - 4 = 0$

$x = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 3 \cdot (-4)}}{2 \cdot 3} = \frac{-6 \pm \sqrt{84}}{6}$



$\tan 50^\circ = \frac{x}{30}$

$x = 30 \tan 50^\circ = 35,7526... \approx 36$

$v: 36 \text{ m}$

7. aika sähköpostin saajia

0 $a_1 = 2$

10 $a_2 = 2 \cdot 2 = 4$

20 $a_3 = 4 \cdot 2 = 8$

30 $8 \cdot 2 = 16$

yht.

$2 + 4 + 8 + 16 ...$

geom. summa $q = 2$

$S = \frac{2(1-2^n)}{1-2} \geq 20000 \quad | :(-2)$

$1-2^n \leq -10000$

$-2^n \leq -10001$

$2^n \geq 10001 \quad | \lg$

$n \lg 2 \geq \lg 10001$

$n \geq \frac{\lg 10001}{\lg 2} = 13,287... \Rightarrow n \geq 14$

Aikaa kului $(n-1) \cdot 10 \text{ min} = 13 \cdot 10 \text{ min} = 130 \text{ min}$
 $= 2 \text{ h } 10 \text{ min}$

8. Korotus 2,5% \Rightarrow kerroin $(100+25)\% = 1,025$

$$45 \cdot 1,025^{10} e = 57,603... e \approx \underline{57,60 e}$$

9. $V_1 = a_1^2 h = 10.000 m^3$
särmä lyhenee 10% \Rightarrow kerroin $(100-10)\% = 0,9$

Särmät $a_1, 0,9a_1, 0,9^2a_1, 0,9^3a_1, \dots$
geom. jono, $q = 0,9$

$$\begin{aligned} V &= a_1^2 h + (0,9a_1)^2 h + (0,9^2a_1)^2 h + (0,9^3a_1)^2 h + \dots \\ &= a_1^2 h + 0,9^2 a_1^2 h + 0,9^4 a_1^2 h + 0,9^6 a_1^2 h + \dots \\ &= 10.000 + 0,9^2 \cdot 10.000 + 0,9^4 \cdot 10.000 + \dots \end{aligned}$$

geom. summa, $q = 0,9^2 = 0,81$

$$V = \frac{10.000 (1 - 0,81^{100})}{1 - 0,81} = 52631,578...$$

$$V \approx \underline{52600 m^3}$$

10. $1,44 M = \log_{10} E - 5,24$

a) $1,44 \cdot 9,0 = \log E_s - 5,24$
 $\log E_s = 12,96 + 5,24$
 $\log E_s = 18,20 \quad | \cdot 10^2$
 $E_s = 10^{18,20} = 1,5848... \cdot 10^{18} \approx \underline{1,6 \cdot 10^{18}}$

b) $1,44 \cdot 6,8 = \log E_k - 5,24$
 $\log E_k = 15,032$
 $E_k = 10^{15,032} = 1,076... \cdot 10^{15}$

$$\frac{E_s}{E_k} = \frac{10^{18,20}}{10^{15,032}} = 10^{18,20 - 15,032} = 10^{3,168} = 1472,312...$$

V: n. 1500 -kertainen

$$\text{tai } \frac{1,5848... \cdot 10^{18}}{1,076... \cdot 10^{15}} = 1472,3...$$

11.

	Asc. (x)		Surs. (y)	
lepakon siipi	$3x$	+	$4y$	≥ 20
hämähäkin seitti	$2x$	+	y	≥ 10
	$x \geq 0$			
	$y \geq 0$			

$$3x + 4y \geq 20$$

$$4y \geq 20 - 3x \quad | :4$$

$$y \geq 5 - \frac{3}{4}x \quad \text{yläpuoli}$$

$$\text{rajasuora } y = 5 - \frac{3}{4}x$$

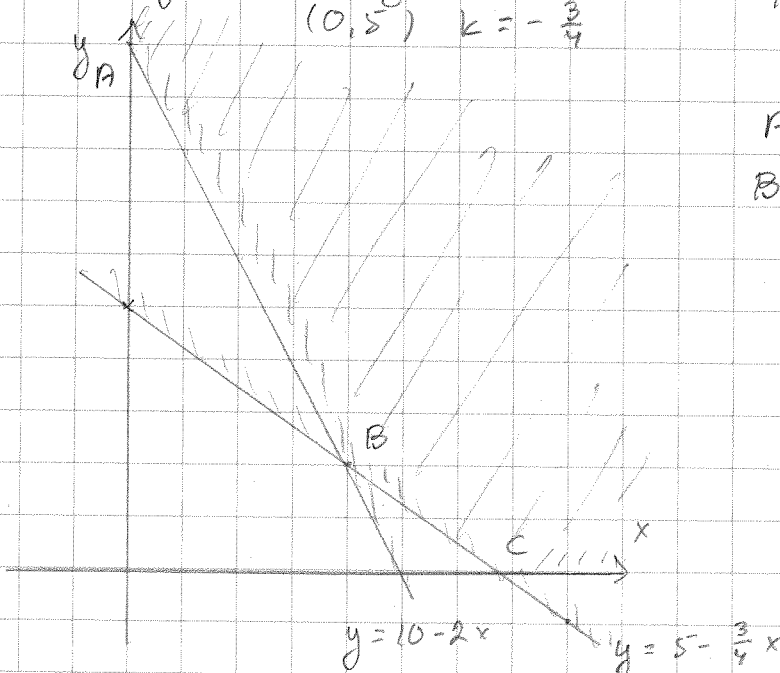
$$(0, 5) \quad k = -\frac{3}{4}$$

$$2x + y \geq 10$$

$$y \geq 10 - 2x \quad \text{yläpuoli}$$

$$\text{rajasuora } y = 10 - 2x$$

$$(0, 10) \quad k = -2$$



$$A = (0, 10)$$

$$B: \begin{cases} y = 5 - \frac{3}{4}x \\ y = 10 - 2x \end{cases}$$

$$5 - \frac{3}{4}x = 10 - 2x$$

$$\frac{5}{4}x = 5$$

$$x = 4$$

$$y = 10 - 2 \cdot 4 = 2$$

$$B = (4, 2)$$

$$C: \begin{cases} y = 0 \\ 3x + 4y = 20 \end{cases}$$

$$3x = 20$$

$$x = 6\frac{2}{3}$$

$$C = (6\frac{2}{3}, 0)$$

hinta $h(x, y) = 2x + 3y$
 pienin arvo jossain alueen
 kärkipisteessä

$$f(0, 10) = 2 \cdot 0 + 3 \cdot 10 = 30$$

$$f(4, 2) = 2 \cdot 4 + 3 \cdot 2 = 14$$

$$f(6\frac{2}{3}, 0) = 2 \cdot 6\frac{2}{3} + 3 \cdot 0 = 13\frac{1}{3} \quad \text{edullisin}$$

V: $6\frac{2}{3}$ pilarillista Ascensusta (ei Sursumia)

MA.
K12

12. $f_1 = f_2 = 1, f_{n+2} = f_{n+1} + f_n, n = 1, 2, \dots$
(2 edellistä)

a) $1, 1, 2, 3, 5, 8, 13, 21, 34, 55$

$f_3 = 1 + 1 = 2$

$f_7 = 5 + 8 = 13$

$f_4 = 1 + 2 = 3$

$f_8 = 8 + 13 = 21$

$f_5 = 2 + 3 = 5$

$f_9 = 13 + 21 = 34$

$f_6 = 3 + 5 = 8$

$f_{10} = 21 + 34 = 55$

b) $\varphi = \frac{1}{2}(1 + \sqrt{5}) = \frac{1 + \sqrt{5}}{2}$

$f_1 = \frac{1}{\sqrt{5}} \left(\frac{1 + \sqrt{5}}{2} - \left(-\frac{1 + \sqrt{5}}{2} \right)^{-1} \right)$

$= \frac{1}{\sqrt{5}} \left(\frac{1 + \sqrt{5}}{2} + \frac{1 - \sqrt{5}}{2} \right) = \frac{1}{\sqrt{5}} \left(\frac{1 + \sqrt{5}}{2} + \frac{2 - 2\sqrt{5}}{2} \right)$

$= \frac{1}{\sqrt{5}} \left(\frac{1 + \sqrt{5}}{2} - \frac{2 - 2\sqrt{5}}{2} \right) = \frac{1}{\sqrt{5}} \cdot \frac{2 + 2\sqrt{5} - 2 + 2\sqrt{5}}{2}$

$= \frac{4\sqrt{5}}{4\sqrt{5}} = 1$

$f_2 = \frac{1}{\sqrt{5}} \left(\left(\frac{1 + \sqrt{5}}{2} \right)^2 - \left(-\frac{1 + \sqrt{5}}{2} \right)^{-2} \right)$

$= \frac{1}{\sqrt{5}} \left(\frac{1 + 2\sqrt{5} + 5}{4} - \frac{6 - 2\sqrt{5}}{4} \right)$

$= \frac{1}{\sqrt{5}} \left(\frac{6 + 2\sqrt{5}}{4} - \frac{24 - 8\sqrt{5}}{36 - 20} \right)$

$= \frac{1}{\sqrt{5}} \cdot \frac{24 + 8\sqrt{5} - 24 + 8\sqrt{5}}{16} = \frac{16\sqrt{5}}{16\sqrt{5}} = 1 \quad \square$

c) $x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{1 \pm \sqrt{5}}{2}$

$x_1 = \frac{1 + \sqrt{5}}{2} = \varphi$

$x_2 = \frac{1 - \sqrt{5}}{2} = \frac{1 - 5}{2(1 + \sqrt{5})} = -\frac{4}{2(1 + \sqrt{5})} = -\frac{2}{1 + \sqrt{5}}$

$= -\frac{1}{\varphi} \quad \square$

$$13. a) \text{ Voitto } 42000 - 12000 - 4000 = 26000$$

$$\text{Vero } 0,3 \cdot 26000 = 7800$$

$$\text{Käteen } 42000 - 7800 = 34200$$

Hankintameno-olettama 40%

$$\text{Voitto } (100 - 40)\% = 60\%$$

$$\text{Vero } 0,3 \cdot 0,6 \cdot 42000 = 7560$$

$$\text{Käteen } 42000 - 7560 = 34440 > 34200$$

V: Käteen jää 34440e

b) myyntihinta x + vero yhtä suuri

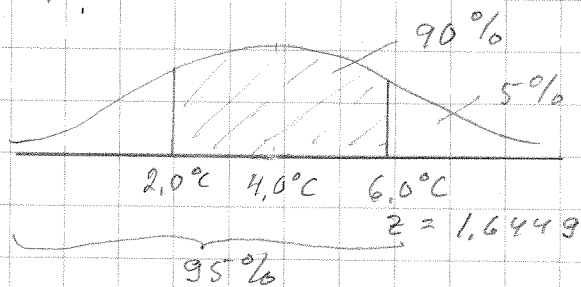
$$x - 0,3(x - 16000) = x - 0,3 \cdot 0,6x$$

$$x - 0,3x + 4800 = x - 0,18x$$

$$0,12x = 4800$$

$$\underline{x = 40000 \text{ (e)}}$$

$$14. \mu = 4,0^\circ\text{C} (= \bar{x})$$



$$\frac{100 - 90}{2} = 5$$

$$\Phi(1,6449) = 0,95$$

$$\frac{6,0^\circ - 4,0^\circ}{\sigma} = 1,6449$$

$$2^\circ = 1,6449\sigma$$

$$\sigma = \frac{2^\circ}{1,6449} = 1,2158...^\circ \approx \underline{\underline{1,2^\circ}}$$

$$15 a) \sin(2x + 4^\circ) = \frac{\sqrt{3}}{2}$$

$$2x + 4^\circ = 60^\circ \quad \vee \quad 2x + 4^\circ = 120^\circ$$

$$2x = 56^\circ$$

$$\underline{x = 28^\circ}$$

taulukosta $60^\circ, 120^\circ$

$$2x = 116^\circ$$

$$\underline{x = 58^\circ}$$

$$x \in [0, 90^\circ]$$

$$b) 2x + 4^\circ = 60^\circ + n \cdot 360^\circ \quad \vee \quad 2x + 4^\circ = 120^\circ + n \cdot 360^\circ$$

$$2x = 56^\circ + n \cdot 360^\circ$$

$$\underline{x = 28^\circ + n \cdot 180^\circ} \quad \vee$$

$$2x = 116^\circ + n \cdot 360^\circ$$

$$\underline{x = 58^\circ + n \cdot 180^\circ}$$

$$\begin{aligned}
 15c) &= \frac{1}{r_1 r_2} \left[r_1^2 + r_2^2 + \sqrt{r_1}^4 + 4\sqrt{r_1}^3 \sqrt{r_2} + 6\sqrt{r_1}^2 \sqrt{r_2}^2 + 4\sqrt{r_1} \sqrt{r_2}^3 + \sqrt{r_2}^4 + 2r_1 r_2 + 2r_1 (r_2 + 2\sqrt{r_1} r_2 + r_1) + 2r_2 (r_1 + 2\sqrt{r_1} r_2 + r_2) \right] \text{ MAX} \\
 &= \frac{1}{r_1^2 r_2^2} \left[r_1^2 + r_2^2 + r_1^2 + 4r_1 \sqrt{r_1} r_2 + 6r_1 r_2 + 4r_2 \sqrt{r_1} r_2 + r_2^2 + 2r_1 r_2 + \right. \\
 &\quad \left. + 2r_1^2 + 4r_1 \sqrt{r_1} r_2 + 2r_1 r_2 + 2r_1 r_2 + 4r_2 \sqrt{r_1} r_2 + 2r_2^2 \right] \\
 &= \frac{4r_1^2 + 8r_1 \sqrt{r_1} r_2 + 12r_1 r_2 + 8r_2 \sqrt{r_1} r_2 + 4r_2^2}{r_1^2 r_2^2}
 \end{aligned}$$

$$2 \left(k_1^2 + k_2^2 + k_3^2 \right) = 2 \left(\frac{r_1}{r_1^2} + \frac{r_2}{r_2^2} + \frac{1}{r_3^2} \right)$$

$$= 2 \left(\frac{r_2^2}{r_1^2 r_2^2} + \frac{r_1^2}{r_1^2 r_2^2} + \frac{(\sqrt{r_1} + \sqrt{r_2})^4}{r_1^2 r_2^2} \right)$$

$$= 2 \frac{r_1^2 + r_2^2 + r_1^2 + 4r_1 \sqrt{r_1} r_2 + 6r_1 r_2 + 4r_2 \sqrt{r_1} r_2 + r_2^2}{r_1^2 r_2^2}$$

$$= \frac{4r_1^2 + 8r_1 \sqrt{r_1} r_2 + 12r_1 r_2 + 8r_2 \sqrt{r_1} r_2 + 4r_2^2}{r_1^2 r_2^2}$$

$$= (k_1 + k_2 + k_3)^2 \quad \square$$