

$$1. a) \begin{aligned} 2(x+4) - 3(x-3) &= 0 \\ 2x+8 - 3x+9 &= 0 \\ -x &= -17 \\ x &= 17 \end{aligned}$$

MAB
K-13

$$b) \frac{3}{4} \text{ ja } \frac{5}{6} \text{ k\ae}n\text{teisluvut } \frac{4}{3} \text{ ja } \frac{5}{6}$$

$$\frac{\frac{4}{3} + \frac{5}{6}}{2} = \frac{\frac{8}{6} + \frac{5}{6}}{2} = \frac{\frac{13}{6}}{2} = \frac{13}{6 \cdot 2} = \frac{13}{12}$$

$$c) \frac{3a - 6a^2}{3a} = \frac{3a}{3a} - \frac{6a^2}{3a} = \underline{\underline{1 - 2a}}$$

$$2a) \begin{aligned} 4x + 17 &> 2 - x \\ 5x &> -15 \quad || :5 \\ x &> -3 \end{aligned}$$

$$b) \begin{aligned} x^2 + 14x &= -49 \\ x^2 + 14x + 49 &= 0 \\ x &= \frac{-14 \pm \sqrt{14^2 - 4 \cdot 1 \cdot 49}}{2 \cdot 1} = \frac{-14 \pm 0}{2} = \underline{\underline{-7}} \end{aligned}$$

$$c) (0,0) \quad (2,3)$$

$$k = \frac{3-0}{2-0} = \frac{3}{2}$$

$$y - 0 = \frac{3}{2}(x - 0) \Leftrightarrow y = \frac{3}{2}x$$

$$(48, 75) \text{ vaatimus: } y = \frac{3}{2} \cdot 48 = 72$$

$$75 \neq 72$$

\Rightarrow E: kulje pisteen kautta

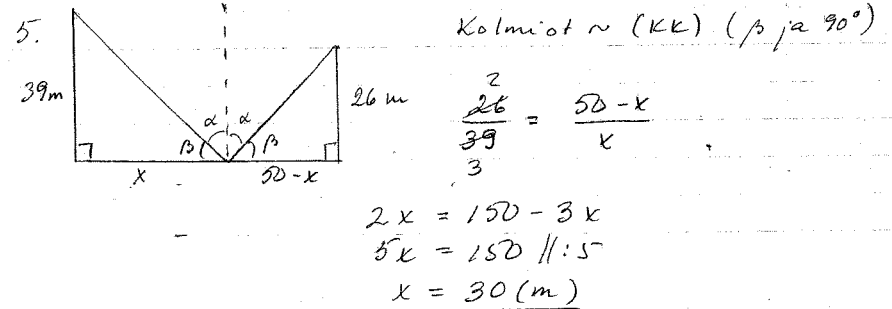
$$3a) \begin{aligned} f(x) &= x(x+2) - 5 = x^2 + 2x - 5 \\ f'(x) &= 2x + 2 \\ f'(1) &= 2 \cdot 1 + 2 = \underline{\underline{4}} \end{aligned}$$

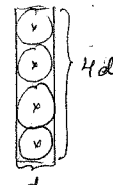
$$b) \begin{aligned} 5^{3x-1} &= 25^{\frac{x}{2}} \\ 5^{3x-1} &= (5^2)^{\frac{x}{2}} \\ 5^{3x-1} &= 5^x \\ 3x-1 &= x \\ 2x &= 1 \quad \Rightarrow x = \frac{1}{2} \end{aligned}$$

$$4. \begin{array}{ll} \text{I osuus} & 21,90 \\ \text{II} & 28,20 - 21,90 = 6,30 \\ \text{III} & 33,50 - 28,20 = 5,30 \end{array}$$

MA
K-1.

$$\begin{aligned} \text{Alpo: } & \frac{1}{3} \cdot 21,90 = 7,30 \text{ (e)} \\ \text{Sanna: } & \frac{1}{3} \cdot 21,90 + \frac{1}{2} \cdot 6,30 = 10,45 \text{ (e)} \\ \text{Pauli: } & \frac{1}{3} \cdot 21,90 + \frac{1}{2} \cdot 6,30 + 5,30 = 15,75 \text{ (e)} \end{aligned}$$



6. 

$$d = 6,68 \text{ cm}$$

$$r = \frac{1}{2}d = 3,34 \text{ cm}$$

$$V_P = 4 \cdot \frac{4}{3}\pi r^3 = \frac{16}{3}\pi r^3 \quad (4 \cdot 156,07... = 624,2)$$

$$V_L = \pi r^2 \cdot 8r = 8\pi r^3 \quad (936,438...)$$

$$\frac{V_P}{V_L} = \frac{\frac{16}{3}\pi r^3}{8\pi r^3} = \frac{2}{3} = 0,666... \approx \underline{\underline{67\%}}$$

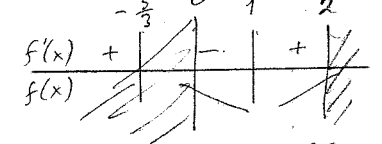
7. $f(x) = 2x^3 + 2x^2 - 10x + 5 \quad [0,2]$

$$f'(x) = 6x^2 + 4x - 10 = 0$$

$$3x^2 + 2x - 5 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4 \cdot 3 \cdot (-5)}}{2 \cdot 3} = \frac{-2 \pm \sqrt{64}}{6} = \frac{-2 \pm 8}{6}$$

$$x = 1 \quad \vee \quad x = -\frac{5}{3}$$



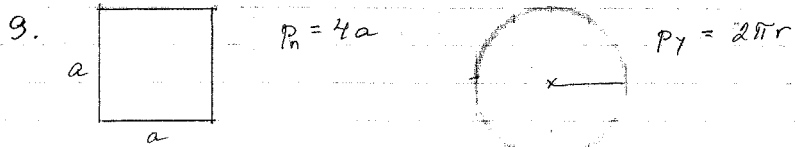
Pienin arvo $f(1) = 2 \cdot 1^3 + 2 \cdot 1^2 - 10 \cdot 1 + 5 = -1$

Suurin joko $f(0) = 5$
tai $f(2) = 2 \cdot 2^3 + 2 \cdot 2^2 - 10 \cdot 2 + 5 = 9$

Funktio saa kaikki arvot v\ae}lilt\ae} [-1, 9]

8. a) $\frac{1460580}{422580} = 3,456... \approx 346\%$ MAB
K-13
 Kasvu oli $(346-100)\% = \underline{246\%}$

b) $1460580 \cdot x^4 = 422580 \parallel : 1460580$
 $x^4 = 0,28928... \parallel \sqrt[4]{\quad}$
 $x = \pm 0,7333... \approx 73,3\%$
 Vähennys $(100-73,3)\% = \underline{26,7\%}$ vuodessa



$4a = 2\pi r \parallel : 4$
 $a = \frac{1}{2}\pi r$
 $A_n = a^2 = (\frac{1}{2}\pi r)^2 = \frac{1}{4}\pi^2 r^2, A_\gamma = \pi r^2$

a) $\frac{A_n}{A_\gamma} = \frac{\frac{1}{4}\pi^2 r^2}{\pi r^2} = \frac{1}{4}\pi = 0,7853...$

$1 - 0,7853... = 0,2146... \approx \underline{21,5\%}$ pienempi

b) $\frac{A_\gamma}{A_n} = \frac{\pi r^2}{\frac{1}{4}\pi^2 r^2} = \frac{1}{\frac{1}{4}\pi} = \frac{4}{\pi} = 1,2732...$

V: 27,3% suurempi

10.6

7	8	9	10	11	12
5	6	7	8	9	10
4	5	6	7	8	9
3	4	5	6	7	8
2	3	4	5	6	7
1	2	3	4	5	6
1	2	3	4	5	6

summa

6	6	12	18	24	30	36
5	5	10	15	20	25	30
4	4	8	12	16	20	24
3	3	6	9	12	15	18
2	2	4	6	8	10	12
1	1	2	3	4	5	6
1	2	3	4	5	6	

a) $P(\text{summa} \geq 8) = \frac{15}{36} = \frac{5}{12} (\approx 0,42)$ ○

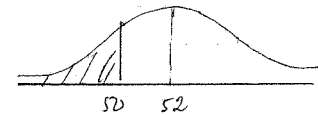
b) $P(\text{summa} > \text{tulo}) = \frac{11}{36} (\approx 0,31)$ x

11. $a_1 = 2, a_2 = \frac{12}{5}$ M1
K.

a) aritm. jono $d = \frac{12}{5} - 2 = \frac{2}{5}$
 $n = 100$
 $a_n = a_1 + (n-1)d$
 $a_{100} = 2 + 99 \cdot \frac{2}{5} = 41 \frac{3}{5}$
 $S_{100} = 100 \cdot \frac{2 + 41 \frac{3}{5}}{2} = \underline{2180}$

b) geom. jono $q = \frac{12/5}{2} = \frac{6}{5}$
 $S_n = \frac{a_1(1-q^n)}{1-q}$
 $S_{100} = \frac{2 \cdot (1 - (\frac{6}{5})^{100})}{1 - \frac{6}{5}} = 828179735,2... \approx \underline{8,28 \cdot 10^8}$

12. $x \sim N(52 \text{ ml}; 1,25 \text{ ml})$ $\bar{x} = 52 \text{ ml}$
 $s = 1,25 \text{ ml}$



$z = \frac{50 - 52}{1,25} = -1,6$

$P(x < 50 \text{ ml}) = P(z < -1,6) = 1 - \Phi(1,6)$
 $= 1 - 0,9452 = 0,0548 \approx \underline{5,5\%}$

13. $N(0) = 2300$ (v. 1971)
 $N(40) = 260000000$ (v. 2011) $N(t) = N(0)e^{at}$

a) $N(40) = N(0)e^{a \cdot 40}$
 $2300 \cdot e^{40a} = 260000000 \parallel : 2300$
 $e^{40a} = 1,13... \cdot 10^6 \parallel \ln$
 $40a = \ln 1,13... \cdot 10^6$
 $a = \frac{\ln 1,13... \cdot 10^6}{40} = 0,348... \approx 0,35$

b) $N(0) \cdot e^{at} = N(0) \cdot 2$
 $e^{at} = 2 \parallel \ln$
 $at = \ln 2$
 $t = \frac{\ln 2}{a} \approx \frac{\ln 2}{0,35} = 1,980... \approx 2$ (vuotta) □

14. x lepl MAB
a) Kulut $98\,000 + 12,30x$ K-13

b) Voitto $17,99 \cdot 0,75x + 14,00 \cdot 0,25x - (98\,000 + 12,30x)$
 $= 13,4925x + 3,5x - 98\,000 - 12,30x$
 $= \underline{\underline{4,6925x - 98\,000}}$

c) $4,6925x - 98\,000 \geq 0$
 $4,6925x \geq 98\,000 \quad || : 4,6925$
 $x \geq 20884,38\dots$

V: väh. 20885 lepl

15. $f(x) = A \sin(bx)$

a) $-1 \leq \sin(bx) \leq 1$ ja $-3 \leq f(x) \leq 3$
 $\Rightarrow \underline{A = 3}$

b) 360° : en. mahtuu puoli "kiemuraa"
eli jaksosa $\Rightarrow \underline{b = 0,5}$

c) Koko "kiemura" vaatii 720°
 $\Rightarrow \underline{\underline{jaksosa 2 = 720^\circ}}$