

1. a)  $4x^2 + 9 = -12x$   
 $4x^2 + 12x + 9 = 0$   
 $x = \frac{-12 \pm \sqrt{144 - 4 \cdot 4 \cdot 9}}{2 \cdot 4} = \frac{-12 \pm \sqrt{0}}{8} = \frac{-12}{8} = \underline{\underline{-\frac{3}{2}}}$

b)  $x = \frac{x^2 + 3}{x-1} \quad | \cdot (x-1)$   
 $x^2 - x = x^2 + 3$   
 $-x = 3$   
 $x = \underline{\underline{-3}}$

c)  $\frac{5x+3y}{3} + \frac{x-6y}{2} = \frac{10x+6y}{6} + \frac{3x-18y}{6} = \frac{13x-12y}{6}$

2. a)  $(-2, 0) \quad (1, 7)$   
 $k = \frac{7-0}{1-(-2)} = \frac{7}{3}$   
 $y-0 = \frac{7}{3}(x+2)$   
 $y = \frac{7}{3}x + \frac{14}{3}$

b)  $\frac{4}{3}\pi r^3 = 1000 \text{ m}^3 \quad | : (\frac{4}{3}\pi)$   
 $r^3 = \frac{1000}{\frac{4}{3}\pi} \text{ m}^3 = 238,732... \text{ m}^3 \quad | \sqrt[3]{\quad}$   
 $r = 6,208... \text{ m} \approx \underline{\underline{6,20 \text{ m}}}$

c)  $2^x = 1024$  tak  $2^x = 1024 \quad | \lg$   
 $2^x = 2^{10}$   $x \lg 2 = \lg 1024$   
 $x = \frac{\lg 1024}{\lg 2} = \underline{\underline{10}}$

3. a) A, B, C, D  $4l = 24$  eri jalgj.

b) 

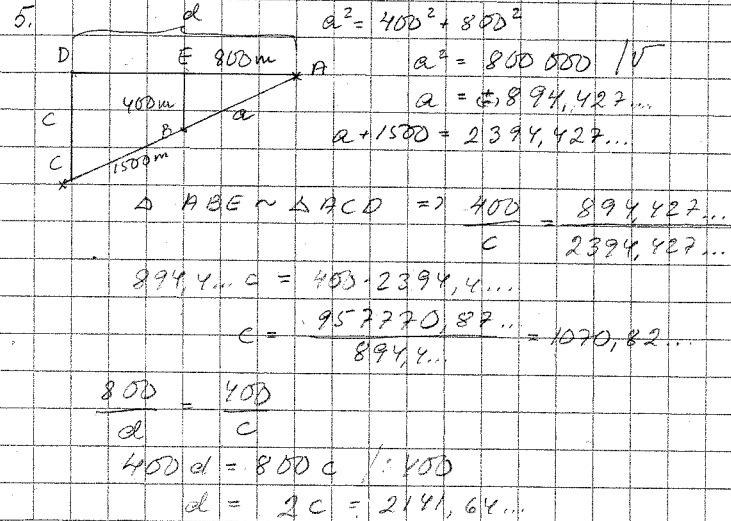
x	y
2	3
5	y

 leiamtään verr.  
 $\frac{2}{5} = \frac{y}{3}$   
 $5y = 6 \quad | : 5$   
 $y = \underline{\underline{\frac{6}{5}}}$

4.

pottoaine	0,35a	x
muu	0,65a	0,65a
kaust.	a	1,10a

$x + 0,65a = 1,10a$   
 $x = 1,10a - 0,65a = 0,45a$   
 $\frac{0,45a}{0,35a} = 1,2857... \approx 128,6\%$   
 $128,6 - 100 = 28,6 \quad \underline{\underline{V: 28,6\%}}$



Lüisa:  $800 \text{ m} + 400 \text{ m} + 1500 \text{ m} = 2700 \text{ m}$   
 $\frac{2700 \text{ m}}{20 \text{ min } 30 \text{ s}} = \frac{2,7 \text{ km}}{\frac{20}{60} + \frac{30}{3600} \text{ h}} = 7,902... \frac{\text{km}}{\text{h}} \approx \underline{\underline{7,9 \frac{\text{km}}{\text{h}}}}$

Pia:  $d + c = 3c = 3212,46... \text{ m} \approx 3200 \text{ m}$   
 $\frac{3212,4... \text{ km}}{0,3972... \text{ h}} = 8,0873... \frac{\text{km}}{\text{h}} \approx \underline{\underline{8,1 \frac{\text{km}}{\text{h}}}}$   
 $20 \text{ min } 30 \text{ s} + 3 \text{ min } 20 \text{ s} = 23 \text{ min } 50 \text{ s} = 0,3972... \text{ h}$

	I sec x	II sec y	
6.	kuparia	$0,25x +$	$0,8y$
	nikkeliä	$0,25x +$	$0,2y = 0,22 \cdot 300g = 66g$
	yhteensä	$x + y$	$= 300g$

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$$\begin{cases} 0,25x + 0,2y = 66 \\ x + y = 300 \end{cases} \cdot (-0,2)$$

$$\begin{cases} 0,25x + 0,2y = 66 \\ -0,2x - 0,2y = -60 \end{cases}$$

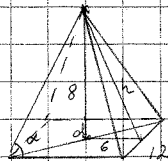
$$0,05x = 6 \quad | \cdot 0,05$$

$$x = 120$$

$$120 + y = 300 \Rightarrow y = 180$$

V: 120g ensimmäistä ja 180g toista

7.



$$h^2 = 8^2 + 6^2$$

$$h^2 = 100 \quad | \sqrt{\quad}$$

$$h = 10$$

$$A_{pinta} = \frac{1}{2} \cdot 12 \cdot 10 = 60$$

$$d^2 = 12^2 + 12^2$$

$$d^2 = 288 \quad | \sqrt{\quad}$$

$$d = 16,970...$$

$$\tan \alpha = \frac{8}{\frac{1}{2}d} = \frac{8}{8,48...}$$

$$\alpha = 43,31^\circ$$

8.

$$y = -3x + 2$$

x-akseli,  $y = 0$

$$-3x + 2 = 0$$

$$-3x = -2$$

$$x = \frac{2}{3}$$

$$y = ax + 6$$

$y = 0$

$$ax + 6 = 0$$

$$ax = -6$$

$$x = -\frac{6}{a}$$

$10 - \frac{6}{a}$  suurempi:

$$-\frac{6}{a} - \frac{2}{3} = 3$$

$$\frac{-18 - 2a}{3a} = 3$$

$$-18 - 2a = 9a$$

$$11a = -18$$

$$a = -\frac{18}{11} = -1\frac{7}{11}$$

$20 - \frac{6}{a}$  pienempi:

$$\frac{2}{3} + \frac{6}{a} = 3$$

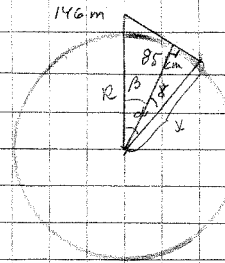
$$\frac{2a + 18}{3a} = 3$$

$$2a + 18 = 9a$$

$$-7a = -18 \Rightarrow a = \frac{18}{7} = 2\frac{4}{7}$$

Vast:  $a = 1\frac{7}{11}$  tai  $a = -2\frac{4}{7}$

9.



$$\frac{85 \text{ km}}{40500 \text{ km}} = \frac{\alpha}{360^\circ}$$

$$40500 \alpha = 360^\circ \cdot 85$$

$$\alpha = \frac{30600^\circ}{40500} = 0,765^\circ$$

$$2\pi R = 40500$$

$$R = \frac{40500}{2\pi} = \frac{20250}{\pi} = 6366,19...$$

$$R + 146 \text{ m} = (6366,19... + 0,146) \text{ km} = 6366,343... \text{ km}$$

$$\cos \beta = \frac{R}{R + 146 \text{ m}}$$

$$\beta = 0,388...^\circ$$

$$\gamma = \alpha - \beta = 0,376...^\circ$$

$$\cos \gamma = \frac{R}{x}$$

$$x \cos \gamma = R$$

$$x = \frac{R}{\cos \gamma} = \frac{6366,197...}{\cos 0,376...} = 6366,335...$$

$$x - R = 0,1377... \text{ km} \approx 138 \text{ m}$$

10.  $2; 1,05 \cdot 2; 1,05^2 \cdot 2; \dots$  geom. jono,  $q = 1,05$

$$a_n = 2 \cdot 1,05^{n-1}$$

$$2 \cdot 1,05^{n-1} < 1000 \cdot 10^6 / 1,2$$

$$1,05^{n-1} < 500 \cdot 10^6 / 1,2$$

$$(n-1) \cdot \lg 1,05 < \lg (500 \cdot 10^6) \quad | : \lg 1,05 > 0$$

$$n-1 < \frac{\lg (500 \cdot 10^6)}{\lg 1,05} (= 410,53...)$$

$$n < 411,53...$$

V: 411 kpl

$$S_{411} = \frac{2 \cdot (1 - 1,05^{411})}{(1 - 1,05)} = 2,0457... \cdot 10^{10}$$

$$\approx 2,05 \cdot 10^{10}$$

11.  $P(A) = 60\% = 0,6$   $P(\bar{A}) = 1 - 0,6 = 0,4$   
 a)  $P(\bar{A}\bar{A}\bar{A}) = 0,4^3 = 0,064 = 6,4\%$   
 $P(\text{ainakin 1 i kää}) = 1 - P(\bar{A}\bar{A}\bar{A}) = 1 - 0,064 = 0,936$   
 $= 93,6\%$

b) 5 runkkua, jokaisessa  $P(\text{ain. 1}) = 0,936$   
 $P(\text{jok.}) = 0,936^5 = 0,7184... \approx 71,8\%$

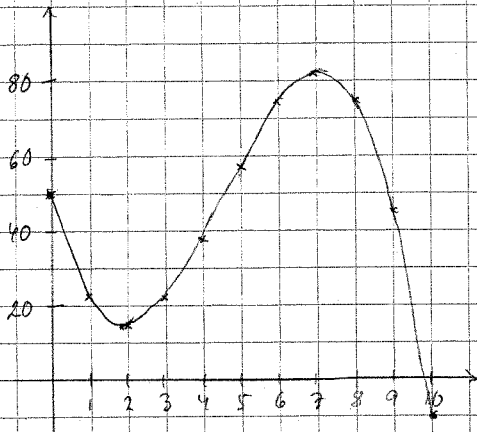
12.  $f(x) = -x^3 + 13,5x^2 - 41x + 50$   $[0, 10]$   
 $f'(x) = -3x^2 + 27x - 41$   
 $-3x^2 + 27x - 41 = 0$   
 $x = \frac{-27 \pm \sqrt{27^2 - 4 \cdot (-3) \cdot (-41)}}{2 \cdot (-3)} = \frac{-27 \pm \sqrt{237}}{-6}$   
 $= \frac{-27 \pm 15,3948...}{-6}$

$x = 1,934... \quad \vee \quad x = 7,0658...$

	0	1,9348...	2,065...	10	
$f'(x)$	/	-	+	+	-
$f(x)$	/	\	/	\	/

$f'(0) = -41$   
 $f'(5) = 19$   
 $f'(10) = -71$

Suurin joko  $f(0) = 50$   
 tai  $f(7,0658...) = 81,5330...$   
V: Suurin arvo 81,533, kun  $x = 7,066$

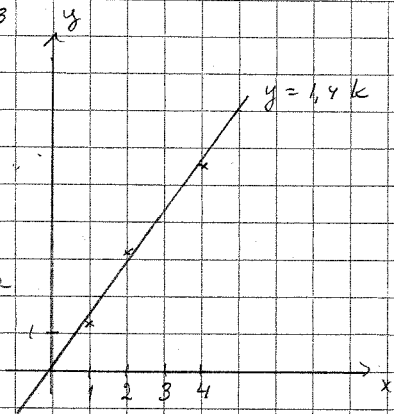


x	y
0	50
1	21,5
min 1,935	13,966...
2	14
3	21,5
4	38
5	57,5
6	74
7	81,5
max 7,066	81,533
8	74
9	45,5
10	-10

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13.  $f(k) = (k-1-1,2)^2 + (k-2-3,1)^2 + (k-4-5,5)^2$   
 $= (k-1,2)(k-1,2) + (2k-3,1)(2k-3,1) + (4k-5,5)(4k-5,5)$   
 $= k^2 - 2 \cdot 1,2k + 1,2^2 + 4k^2 - 2 \cdot 2k \cdot 3,1 + 3,1^2 + 16k^2 - 2 \cdot 4k \cdot 5,5 + 5,5^2$   
 $= 21k^2 - 58,8k + 41,3$   
 $f'(k) = 42k - 58,8 = 0$   
 $42k = 58,8 / : 42$   
 $k = 1,4$

$(k = 1,4 = \frac{7}{5})$   
 $y = kx$  kulkee  
 origon kautta



14. lähdevero 28%  
 tod korke 0,72 · 0,022 = 0,01584

$x \leftarrow \frac{1}{1,01584}$

1v 2500 2500 2500 2500 2500  
 paljonko kullekin 2500:lle piti alenperin sijoittaa (= 2500:n nykyarvo)

$2500 \cdot 1,01584^{-1} + 2500 \cdot 1,01584^{-2} + \dots + 2500 \cdot 1,01584^{-5}$   
 $= 2500 (1,01584^{-1} + 1,01584^{-2} + \dots + 1,01584^{-5})$   
 $= 2500 \cdot 4,770911... = 11927,278... \approx 11927,28(\text{€})$

15. A)  $\bar{x} = 253$  h  $s = 12$  h  $n = 50$   
 95% in luottamusväli  
 alaraja  $253h - 1,96 \cdot \frac{12h}{\sqrt{50}} = 249,67... h \approx 249,7h$   
 yläaraja  $253h + 1,96 \cdot \frac{12h}{\sqrt{50}} = 256,32... h \approx 256,3h$   
V: [249,7h; 256,3h]

Luottamusvälin pituus on  $2 \cdot 1,96 \cdot \frac{12}{\sqrt{n}} = 2 / \sqrt{n}$   
 $\sqrt{n} = 23,52 / ( )^2$   
 $n = 553,19...$   
V: 553 kpl

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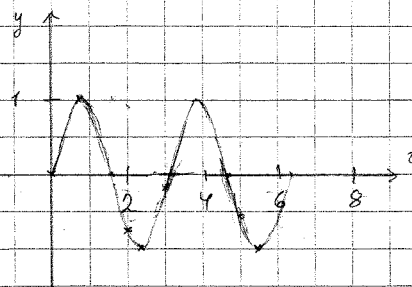
15 B

 $A \sin(bt)$   $t$  (s)MAB  
S08

Huom! Kulma radiaanina

$$A = 1, b = 2$$

$$f(t) = \sin 2t \text{ jaksu on } \frac{2\pi}{2} = \pi$$



jaksu on 3,2 s

$$\frac{2\pi}{b} = 3,2$$

$$3,2 b = 2\pi \quad | : 3,2$$

$$b = \frac{2\pi}{3,2} = 1,963495..$$

$$\approx \underline{\underline{1,96352}}$$

t	y
0	0
$\frac{\pi}{4}$ 0,79	1
1	0,91
$\frac{\pi}{2}$ 1,57	0
2	-0,76
$\frac{3\pi}{4}$ 2,36	-1
3	-0,28
$\pi$ 3,14	0
$\frac{5\pi}{4}$ 3,93	1
4	0,99
$\frac{3\pi}{2}$ 4,71	0
5	-0,54
$\frac{7\pi}{4}$ 5,50	-1
6	-0,54
$2\pi$ 6,28	0