

$$1. a) \frac{1-5x}{5} = \frac{3+4x}{2}$$

$$15+20x = 2-10x$$

$$30x = -13$$

$$x = -\frac{13}{30}$$

$$b) (5+x^2) - (4x^2+x) = 5+x^2-4x^2-x = 5-x-3x^2$$

$$x = -2 \quad 5 - (-2) - 3 \cdot (-2)^2 = 5+2-3 \cdot 4 = -5$$

$$c) 4x^2 - 8x + 3 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 4 \cdot 4 \cdot 3}}{2 \cdot 4} = \frac{8 \pm 4}{8}$$

$$x = \frac{1}{2} \vee x = \frac{3}{2}$$

$$2. a) \frac{2+5+6+a}{4} = 2 \quad | \cdot 4$$

$$13+a = 8$$

$$a = -5$$

$$b) 5a^2 - (2a)^2 = 5a^2 - 4a^2 = a^2$$

$$c) | -x | = x \quad \text{ei silloin kun } x < 0$$

esim.  $x = -2 \quad | -(-2) | = 2 \neq x$

$$3. f(x) = \frac{1}{3}x^3 - 2x^2 + 4$$

$$a) f'(x) = \frac{1}{3} \cdot 3x^2 - 2 \cdot 2x = x^2 - 4x$$

$$b) x\text{-aks. suunt. } \Leftrightarrow k = 0 \Leftrightarrow f'(x) = 0$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x = 0 \vee x = 4$$

c)	$f'(x)$	+	+	-	+	+	$f'(-1) = (-1)^2 - 4 \cdot (-1) = 5$
	$f(x)$	/	\	/	\	/	$f(1) = 1^2 - 4 \cdot 1 = -3$
							$f(5) = 5^2 - 4 \cdot 5 = 5$

Vähenevä, kun  $0 \leq x \leq 4$

NAB

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4.

$$1,17 \times 54,35 \quad | : 1,17$$

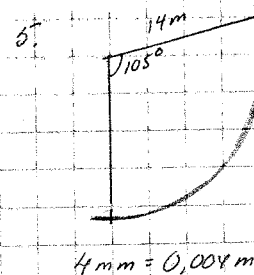
$$x = 46,452... \approx 46,45 \quad \text{veroton hinta}$$

$$17\% - 9\% = 8\%$$

$$1,08 \cdot 46,45 = 50,166 \approx 50,17$$

$$54,35 - 50,17 = 4,18 \quad \text{Hinta alenisi } 4,18 \text{ €}$$

$$\frac{4,18}{54,35} = 0,0769... \quad \text{ja } 7,7\%$$



$$A = \frac{105^\circ}{360^\circ} \cdot \pi \cdot 14^2 \text{ m}^2 = 179,59... \text{ m}^2$$

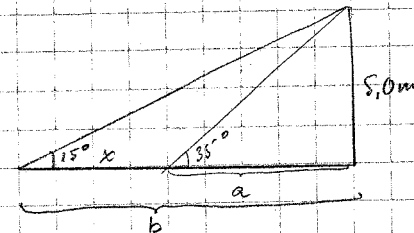
$$V = Ah = 179,59... \cdot 0,004 \text{ m}^3$$

$$= 0,7183... \text{ m}^3 = 718,3... \text{ dm}^3$$

$$\frac{718,3... \text{ dm}^3}{32 \text{ dm}^3/\text{min}} = 22,449... \text{ min}$$

$$\approx 22,4 \text{ min}$$

6.



$$\tan 35^\circ = \frac{5,0}{a}$$

$$a \tan 35^\circ = 5,0$$

$$a = \frac{5,0}{\tan 35^\circ} (= 7,14...)$$

$$\tan 15^\circ = \frac{5,0}{b} \Rightarrow b = \frac{5,0}{\tan 15^\circ} (= 18,66...)$$

$$x = b - a = 11,519... \text{ m} \approx 12 \text{ m}$$

7.

$$1990 \quad 492400 \quad 2000 \quad 555474$$

$$492400 \cdot x^{10} = 555474 \quad | : 492400$$

$$x^{10} = 1,1280... \quad | \sqrt{10}$$

$$x = 1,0121...$$

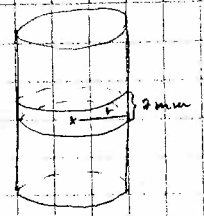
$$555474 \cdot 1,0121...^{15} = 665552,42...$$

V: 1,2% vuodessa, v. 2015 665552 hlöä

NAB

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8.



$2r = 20 \text{ mm}$   
 $r = 10 \text{ mm}$

$V = \pi r^2 \cdot h = 5 \cdot \frac{\pi}{3} \cdot 10^3$   
 $7r^2 = \frac{20000}{3} \quad | : 7$   
 $r^2 = \frac{20000}{21} \quad | \sqrt{\quad}$   
 $r = 30,86... \text{ (mm)}$   
 $2r = 61,72... \text{ mm} \approx \underline{6,2 \text{ cm}}$

MAB  
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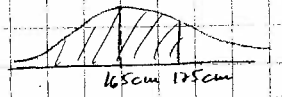
11.

$a_1 = 2$   
 $a_2 = 2 \cdot 2$   
 $a_3 = 2 \cdot (2 \cdot 2) = 2^3$   
 $\vdots$   
 $a_n = 2^n$  geomet. jono,  $q = 2$   
 $S_5 = \frac{2 \cdot (1 - 2^5)}{1 - 2} = \underline{62}$   
 $S_n = \frac{2 \cdot (1 - 2^n)}{1 - 2} = -2(1 - 2^n) = \underline{2 \cdot (2^n - 1)}$

MAB  
 509

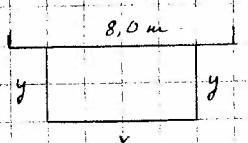
9.

$X \sim N(165 \text{ cm}, 6 \text{ cm})$



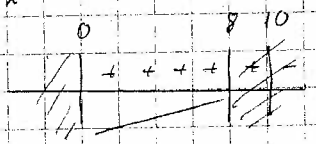
$P(X \leq 175 \text{ cm})$   
 $= P(Z \leq 1,67) = \Phi(1,67) = 0,9525$   $Z = \frac{175 - 165}{6} \approx 1,67$   
 $\approx \underline{0,95}$  (= P(1 tyttö korke 175 cm))  
 $P(\text{käikkien korkeus} \leq 175 \text{ cm}) = 0,9525^3 = 0,86416... \approx \underline{0,86}$   
 $P(\text{ain 1 yli 175 cm}) = 1 - 0,9525^3 = 0,13584... \approx \underline{0,14}$

10.



$x + 2y = 20,0 \text{ m}$   
 $2y = 20 - x$   
 $y = 10 - \frac{1}{2}x$   $0 \leq x \leq 8$

$A(x) = x(10 - \frac{1}{2}x) = 10x - \frac{1}{2}x^2$   
 $A'(x) = 10 - x = 0$   
 $x = 10$



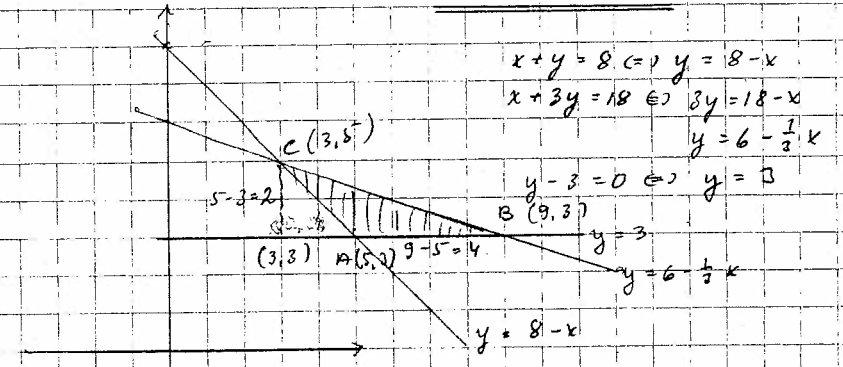
Suurin ala

$A(8) = 10 \cdot 8 - \frac{1}{2} \cdot 8^2 = 80 - 32 = 48 \text{ (m}^2\text{)}$   
 $A'(x) = 10 - x = 9$   
 $A'(4) = 10 - 4 = 6$

$x = 8 \text{ m}$   $y = 10 - \frac{1}{2} \cdot 8 = 6 \text{ (m)}$

$V: 48 \text{ m}^2$ , sivut 8 m ja 6 m.

12.



$V: 13$  vuokkoa

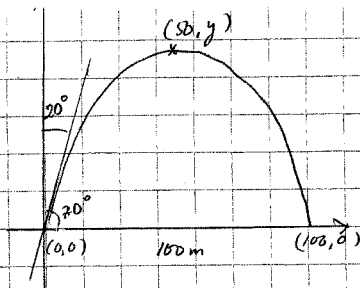
A:  $\begin{cases} y = 3 \\ x + y = 8 \end{cases} \Rightarrow x + 3 = 8 \Rightarrow x = 5 \quad (5, 3)$

B:  $\begin{cases} y = 3 \\ x + 3y = 18 \end{cases} \Rightarrow x + 9 = 18 \Rightarrow x = 9 \quad (9, 3)$

C:  $\begin{cases} x + y = 8 \\ x + 3y = 18 \end{cases} \quad (-1) \quad (3, 5) \quad A = \frac{2 \cdot 4}{2} = 4$

$\begin{cases} -x - y = -8 \\ x + 3y = 18 \end{cases} \quad \begin{matrix} \text{yläpuoli} \\ \text{aläpuoli} \end{matrix} \begin{cases} y \geq 8 - x \\ y \leq 6 - \frac{1}{3}x \end{cases} \quad \text{eli} \quad \begin{cases} x + y \geq 8 \\ x + 3y \leq 18 \end{cases}$   
 $\begin{cases} 2y = 10 \\ y = 5 \end{cases} \quad \begin{matrix} \text{yläpuoli} \\ \text{aläpuoli} \end{matrix} \begin{cases} y \geq 3 \\ y - 3 \geq 0 \end{cases}$

13.



$$k = \tan 80^\circ = 5, \dots = f'(0) \quad \text{MAB} \\ \text{S09}$$

$$f(x) = ax^2 + bx + c$$

$$f(0) = 0 \Leftrightarrow c = 0$$

$$f(100) = 0 \Leftrightarrow a \cdot 100^2 + b \cdot 100 = 0$$

$$f'(x) = 2ax + b$$

$$f'(0) = b = \tan 70^\circ$$

$$10000a + 100 \tan 70^\circ = 0$$

$$10000a = -100 \tan 70^\circ \quad | :10000$$

$$a = -\frac{\tan 70^\circ}{100}$$

$$f(x) = -\frac{\tan 70^\circ}{100} x^2 + \tan 70^\circ x$$

$$\text{huipun } x = \frac{100 - 0}{2} = 50$$

$$y = f(50) = -\frac{\tan 70^\circ}{100} \cdot 50^2 + \tan 70^\circ \cdot 50 = 68,68 \dots \approx 69$$

V: 69 m korkeudella

$$14. \quad \cos 3\alpha = \frac{\sqrt{3}}{2} \quad \text{taulukosta } 30^\circ \quad n \quad 10^\circ + n \cdot 120^\circ \quad -10^\circ + n \cdot 120^\circ \\ 3\alpha = \pm 30^\circ + n \cdot 360^\circ \quad | :3 \quad 1 \quad 150^\circ \quad 110^\circ \\ \alpha = \pm 10^\circ + n \cdot 120^\circ \quad 2 \quad 250^\circ \quad 230^\circ \\ 3 \quad 370^\circ \quad 350^\circ \\ 4 \quad 470^\circ \quad 450^\circ$$

Välillä  $[180^\circ, 360^\circ]$  ovat  
 $230^\circ, 250^\circ$  ja  $350^\circ$

15.	velkaa	korkeo	lyh.	velkaa
	81600	$0,042 \cdot 81600 = 3427,20$	4800	$81600 - 4800 = 76800$
	76800	$0,042 \cdot 76800 = 3225,60$	4800	72000
	;			
	4800	201,60	4800	0

$$\frac{81600}{4800} = 17 \quad \text{lyhennyksi ja vuotta}$$

Laina pienenee  $4800 \text{ e/v}$  (aritm. jono)

Korkeo  $\dots - 0,042 \cdot 4800 \text{ e/v} \dots - d = -201,60$

$$S_{17} = 17 \cdot \frac{3427,20 + 201,60}{2} = \underline{\underline{30544,80 \text{ (e)}}}$$

