MAK
5-11

1. a) $\frac{x^2-2x+1}{x-1}$, $x=3$

$\frac{3^2-2\cdot 3+1}{3-1} = \frac{9-6+1}{2} = \frac{4}{2} = \underline{2}$

b) $\frac{5}{x} = -\frac{1}{2}$

$-x = 10 \quad /\cdot(-1)$

$\underline{x = -10}$

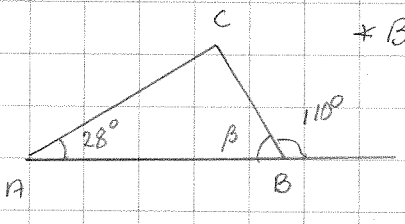
c) $x^2 - 3(x+3) = 3x-18$

$x^2 - 3x - 9 = 3x - 18$

$x^2 - 6x + 9 = 0$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4\cdot 1\cdot 9}}{2\cdot 1} = \frac{6\pm 0}{2}$ $\quad \underline{x=3}$

2. a)



$*B = \beta = 180° - 110° = \underline{70°}$
(vieruskulmien summa on 180°)

$*C = 180° - 28° - 70° = \underline{82°}$
(kolmion kulmien summa on 180°)

b) $\frac{ax}{2} - 1 = \frac{b-2}{2} \quad /\cdot 2 \qquad a \neq 0$

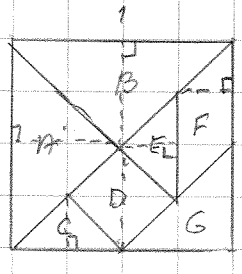
$ax - 2 = b - 2$

$ax = b \quad /:a$

$\underline{x = \frac{b}{a}}$

c) $a^{\frac{1}{2}} b^{\frac{1}{2}} (ab)^{\frac{1}{2}} = a^{\frac{1}{2}} b^{\frac{1}{2}} a^{\frac{1}{2}} b^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} b^{\frac{1}{2}+\frac{1}{2}} = \underline{ab}$

3.



$A = B = \frac{1\cdot\frac{1}{2}}{2} = \frac{1}{4}$ (kolmion $A = \frac{ah}{2}$)

$C = E = \frac{\frac{1}{2}\cdot\frac{1}{4}}{2} = \frac{1}{16}$ —"—

$D = 2\cdot C = \frac{2}{16} = \frac{1}{8}$

$F = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$ (suunn. a/a $A = ah$)

$G = \frac{\frac{1}{2}\cdot\frac{1}{2}}{2} = \frac{1}{8}$

MAE 5-11

4. Beethoven x
Mozart $x-21$ $\quad 56-21=35$
Bach $x+9$ $\quad 56+9=65$

yhteensä $x+x-21+x+9=156$

$3x=156+21-9$

$3x=168 \mid :3$

$x=56$

V: Beethoven 56 v, Mozart 35 v, Bach 65 v

5. Alkuper. arvo a

a) $a\cdot(1-0{,}46)\cdot(1+0{,}15)\cdot(1+0{,}34)$

$=a\cdot 0{,}54\cdot 1{,}15\cdot 1{,}34=0{,}83214\,a<a$

V: Pienempi

b) $a\cdot 0{,}54\cdot 1{,}15\cdot x=a \mid :a$

$0{,}621\,x=1 \mid :0{,}621$

$x=1{,}6103\ldots\approx 161\,\%$ $\quad (161-100)\,\%=61\,\%$

6. $f(x)=x^3-4x+1$ $\quad [-1,2]$

$f'(x)=3x^2-4$

$3x^2-4=0$

$3x^2=4 \mid :3$

$x^2=\frac{4}{3} \mid \sqrt{\ }$

$x=\pm\frac{2}{\sqrt{3}}\ (=\pm 1{,}154\ldots)$

	-1		$-\frac{2}{\sqrt{3}}$		$\frac{2}{\sqrt{3}}$		2
$f'(x)$	$+$	0	$-$	0	$+$	0	$+$
$f(x)$			↘		↗		

$f'(-2)=3\cdot(-2)^2-4=8$

$f'(0)=-4$

$f'(2)=3\cdot 2^2-4=8$

Suurin arvo joko $f(-1)=(-1)^3-4\cdot(-1)+1=4$

tai $f(2)=2^3-4\cdot 2+1=1$

Pienin arvo $f\left(\frac{2}{\sqrt{3}}\right)=\left(\frac{2}{\sqrt{3}}\right)^3-4\cdot\frac{2}{\sqrt{3}}+1=\frac{8}{\sqrt{27}}-\frac{8}{\sqrt{3}}+1$

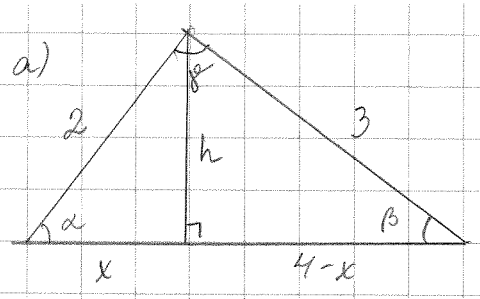
$=\frac{8}{\sqrt{9}\cdot 3}-\frac{3/8}{\sqrt{3}}+1=\frac{8}{3\sqrt{3}}-\frac{24}{3\sqrt{3}}+1$

$=1-\frac{16}{3\sqrt{3}}\approx -2{,}1$

V: Suurin arvo on 4, pienin $1-\frac{16}{3\sqrt{3}}\approx -2{,}1$

MAE
S-11

7. a)



$h^2 + x^2 = 2^2 \Rightarrow h^2 = 4 - x^2$

$h^2 + (4-x)^2 = 3^2$

$h^2 + (4-x)(4-x) = 9$

$h^2 + 16 - 4x - 4x + x^2 = 9$

$h^2 = -x^2 + 8x - 7$

4

$-x^2 + 8x - 7 = 4 - x^2$

$8x = 11$

$x = \frac{11}{8}$

$h^2 = 4 - \left(\frac{11}{8}\right)^2$

$h^2 = \frac{135}{64} \quad | \sqrt{}$

$h = \pm \frac{\sqrt{135}}{8} = 1,452\ldots \approx \underline{1,45}$

b) $\cos\alpha = \frac{\frac{11}{8}}{2} = \frac{11}{16}$

$\alpha = 46,56\ldots^\circ \approx \underline{47^\circ}$

$\cos\beta = \frac{4 - \frac{11}{8}}{3} = \frac{21}{24}$

$\beta = 28,95\ldots^\circ \approx \underline{29^\circ}$

$\gamma = 180^\circ - \alpha - \beta \approx \underline{104^\circ}$

8. $A = 1834\,000\ km^2 = 1,834 \cdot 10^6\ km^2$

$h = 2\ km$

$V = 2 \cdot 1,834 \cdot 10^6\ km^3 = 3,668 \cdot 10^6\ km^3$

sulaa $0,3 \cdot 3,668 \cdot 10^6\ km^3 = 1,1004 \cdot 10^6\ km^3$

$= 1,1004 \cdot 10^{18}\ dm^3$

$1,1004 \cdot 10^{18}\ dm^3 \cdot 0,9\ kg/dm^3 = 0,99036 \cdot 10^{18}\ kg$

Vetenä $0,99036 \cdot 10^{18}\ kg \rightarrow 0,99036 \cdot 10^{18}\ dm^3$

$= 0,99036 \cdot 10^6\ km^3$

Veden pinta $0,71 \cdot 4\pi \cdot (6400\ km)^2 = 3,654\ldots \cdot 10^8\ km^2$

$h_{vesi} = \frac{V_{vesi}}{A_{vesi}} = \frac{0,99036 \cdot 10^6\ km^3}{3,654\ldots \cdot 10^8\ km^2} = 0,002709\ldots\ km$

$\approx \underline{2,7\ m}$

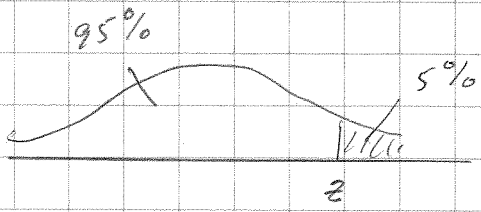
9. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f yht 16

R G B

16·16 16·16 16·16

$16^2 \cdot 16^2 \cdot 16^2 = 16^6 = \underline{16\,777\,216\ erilaista}$

10. $\bar{x} = 30$, $s = 10$



$\Phi(z) = 0{,}95$

taulukosta $z = 1{,}6449$

$$\frac{x-30}{10} = 1{,}6449 \quad | \cdot 10$$

$x - 30 = 16{,}449$

$x = 46{,}449$

osuus $\leq 5\%$ $\Rightarrow$ $x \geq 46{,}449$ p

V: Pisterajä on 47 p

11. $y = x^2 + 4$ $\quad A = (3, 13)$

$y' = 2x$ $\quad k_t = y'(3) = 2 \cdot 3 = 6$

tang. yhtälö $\quad y - 13 = 6(x - 3)$

$y = 6x - 18 + 13$

$y = 6x - 5$

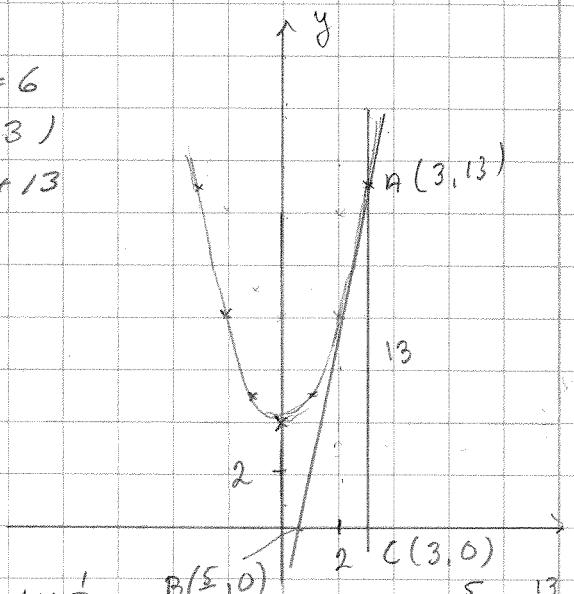
x-akseli. $y = 0$ $\quad 6x - 5 = 0$

$6x = 5$

$x = \frac{5}{6}$

$B = (\frac{5}{6}, 0)$

$C = (3, 0)$



$A = \frac{1}{2} \cdot ah = \frac{1}{2} \cdot \frac{13}{6} \cdot 13 = \frac{169}{12} = 14\frac{1}{12}$

$BC = 3 - \frac{5}{6} = \frac{13}{6}$

11.

x	y
2	1,5
3	2,6
6	4,6

$f(k) = (2k - 1{,}5)^2 + (3k - 2{,}6)^2 + (6k - 4{,}6)^2$

$= 4k^2 - 6k + 2{,}25 + 9k^2 - 15{,}6k + 6{,}76 +$

$+ 36k^2 - 55{,}2k + 21{,}16$

$= 49k^2 - 76{,}8k + 30{,}17$

$f'(k) = 98k - 76{,}8$

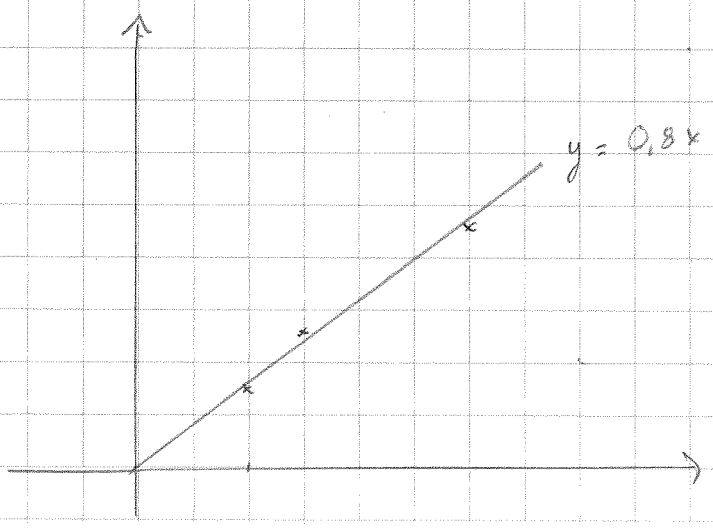
$98k - 76{,}8 = 0$

$98k = 76{,}8 \quad | : 98$

$k = 0{,}783...$

		0,783...	
$f'(k)$	−	0	+
$f(k)$	↘		↗

Pienin arvo, kun $k = 0{,}783... \approx 0{,}8$



13. 13:lla jaolliset luvut

$13, 26, 39, \ldots$ $a_n = 13n$ aritm. jono, $d = 13$

$$S_n = n \cdot \frac{a_1 + a_n}{2}$$

$1000 < 13n < 2000 \quad | : 13$

$76\frac{12}{13} < n < 153\frac{4}{13}$

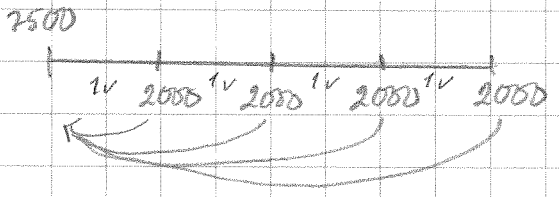
$77 \leq n \leq 153$

Välissä on $153 - 76 = 77$ lukua

$a_{77} = 13 \cdot 77 = 1001$ $\quad a_{153} = 13 \cdot 153 = 1989$

Summa $= 77 \cdot \frac{1001 + 1989}{2} = \underline{\underline{115\,115}}$

14.



$2000 \cdot q^{-1} + 2000 \cdot q^{-2} + 2000 \cdot q^{-3} + 2000 \cdot q^{-4} = 7500 \quad | \cdot q^4$

$2000 \cdot q^3 + 2000 \cdot q^2 + 2000 \cdot q + 2000 = 7500 q^4$

geom. summa

$$2000 \cdot \frac{1 - q^4}{1 - q} = 7500 q^4 \quad | \cdot (1 - q)$$

$2000 \cdot (1 - q^4) = 7500 (q^4 - q^5)$

$2000 - 2000 q^4 = 7500 q^4 - 7500 q^5$

$7500 q^5 - 9500 q^4 + 2000 = 0 \quad | : 500$

$15 q^5 - 19 q^4 + 4 = 0$

Merk. $f(q) = 15 q^5 - 19 q^4 + 4$ 0 on positiivisten ja negatiivisten välissä

$f(1) = 0$ ei käy, ei korkoa

$f(1{,}01) = -0{,}006325$

$f(1{,}02) = -0{,}004999$ $\quad f(1{,}03) = 0{,}004437$

$f(1{,}026) = -0{,}000338$ $\quad f(1{,}027) = 0{,}0007305$

puolivälissä $\quad f(1{,}0265) = 0{,}0001859$

Siis 0-kohta on lähempänä q:n arvoa $1{,}026$ $\quad \underline{V: 2{,}6\,\%}$

Tai: annuiteetti on 2000, $K = 7500$, $n = 4$

$$a = K q^n \frac{1 - q}{1 - q^n} \qquad 2000 = 7500 q^4 \cdot \frac{1 - q}{1 - q^4} \quad | \cdot (1 - q^4)$$

jne.

MAA
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15. $x \in [0°, 360°]$

a) $\sin x = \frac{1}{3}$

Laskimesta $x = 19,47...° \approx \underline{19°}$

toinen vaihtoehto $x = 180° - 19° = \underline{161°}$

b) $\cos x = \frac{1}{4}$

Laskimesta $x = 75,52...° \approx \underline{76°}$

toinen $x = 360° - 76° = \underline{284°}$

c) $\tan x = \frac{1}{5}$

Laskimesta $x = 11,309...° \approx \underline{11°}$

toinen $x = 11° + 180° = \underline{191°}$