

1. a)  $x^2 - 2x = 0$   
 $x(x-2) = 0$   
 $x = 0 \vee x - 2 = 0$   
 $x = 0 \vee x = 2$

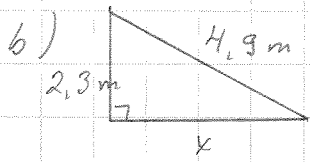
b)  $\frac{2}{3}x - 1 = \frac{2}{3} \quad \parallel \cdot 3$   
 $2x - 3 = 2$   
 $2x = 5 \quad \parallel : 2$   
 $x = 2\frac{1}{2}$

c)  $\begin{cases} x + 2y = -4 \\ 2x - y = -3 \end{cases} \cdot 2 \quad \begin{cases} x + 2y = -4 \\ 4x - 2y = -6 \end{cases}$   
 $5x = -10 \quad \parallel : 5$   
 $x = -2$

$2 \cdot (-2) - y = -3$   
 $-y = -3 + 4$   
 $y = -1$

$V: x = -2, y = -1$

2 a)  $\frac{0,9 \text{ g}}{250 \text{ g}} = 0,036 = \underline{\underline{3,6\%}}$



$x^2 + 2,3^2 = 4,9^2$   
 $x^2 = 4,9^2 - 2,3^2$   
 $x^2 = 18,72 \quad \parallel \sqrt{\quad}$   
 $x = \pm 4,326... \approx 4,3$

$V: 4,3 \text{ m}$

c)  $(0,8) \quad (12,0)$

$k = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 8}{12 - 0} = -\frac{2}{3}$

$y - y_0 = k(x - x_0)$   
 $y - 0 = -\frac{2}{3}(x - 12)$   
 $y = -\frac{2}{3}x + 8$

tai suoraan:  
 $y$  - aks. leikk. piste  $(0,8)$   
 $y = kx + b = -\frac{2}{3}x + 8$

$$3. a) f(x) = x(x+2)^2 = x(x+2)(x+2) = x(x^2 + 2x + 2x + 4) \quad \text{MAA}$$

$$= x^3 + 4x^2 + 4x \quad \text{S 12}$$

$$f'(x) = 3x^2 + 8x + 4$$

$$f'(0) = 3 \cdot 0^2 + 8 \cdot 0 + 4 = \underline{\underline{4}}$$

$$b) \quad 2^{3x+1} = 32$$

$$2^{3x+1} = 2^5$$

$$3x+1 = 5$$

$$3x = 4$$

$$x = \frac{4}{3} = 1\frac{1}{3}$$

$$\text{tai} \quad 2^{3x+1} = 32 \quad \parallel \lg$$

$$(3x+1) \lg 2 = \lg 32 \quad \parallel : \lg 2$$

$$3x+1 = \frac{\lg 32}{\lg 2} (=5)$$

$$3x = 5-1$$

$$x = \frac{4}{3}$$

$$c) \quad \log_4(3x) = 3$$

$$3x = 4^3$$

$$3x = 64 \quad \parallel : 3$$

$$x = 21\frac{1}{3}$$

$$\text{MAOL: } \log_a k = y \Leftrightarrow a^y = k$$

$$\text{eli } x = a^y$$

$$\text{s\u00fcs } 3x = 4^3$$

$$4. \quad y = x^2 - 12x + 35$$

$$a) \quad x\text{-aks. } y = 0$$

$$x^2 - 12x + 35 = 0$$

$$x = \frac{12 \pm \sqrt{12^2 - 4 \cdot 1 \cdot 35}}{2 \cdot 1} = \frac{12 \pm 2}{2}$$

$$x = 7 \quad \vee \quad x = 5$$

V: pisteiss\u00e4 (7,0) ja (5,0)

$$b) \quad \text{huipussa } y' = 0$$

$$y' = 2x - 12 = 0$$

$$2x = 12$$

$$x = 6$$

$$y = 6^2 - 12 \cdot 6 + 35 = -1$$

$$\underline{\underline{V: (6, -1)}}$$

$$5. a) \sum_{n=0}^{22} (3+4n) = (3+4 \cdot 0) + (3+4 \cdot 1) + (3+4 \cdot 2) + \dots + (3+4 \cdot 22)$$

$$= 3 + 7 + 11 + \dots + 91$$

aritm. summa  $d = 4$   $n = 23$  (0 ja 1-22)

$$S_n = n \cdot \frac{a_1 + a_n}{2}$$

$$S_{23} = 23 \cdot \frac{3 + 91}{2} = \underline{\underline{1081}}$$

$$5b) \sum_{n=2}^{15} (-3)^n = (-3)^2 + (-3)^3 + (-3)^4 + \dots + (-3)^{15}$$

$$= 9 - 27 + 81 - \dots - 14348907$$

geom. summa  $q = \frac{-27}{9} = -3, n = 14 (2-15)$

$$S_n = \frac{a_1(1-q^n)}{1-q} = \frac{9(1-(-3)^{14})}{1-3} = \underline{\underline{21523356}}$$

6 Japanilainen  $\frac{6,8 \text{ l}}{100 \text{ km}} = 0,068 \text{ l/km}$

Amerikkalainen  $\frac{1 \text{ gallona}}{32 \text{ mile}} = \frac{3,785 \text{ l}}{32 \cdot 1,609 \text{ km}} = 0,0735 \dots \frac{\text{l}}{\text{km}}$

$\approx 0,074 \text{ l/km} > 0,068 \text{ l/km}$

V: Japanilainen kuluttaa vähemmän

7. a)

	Mercurius	Venus	Maa	Mars	Jupiter
kierroaika $x$	0,241	0,615	1,0	1,881	11,861
$\sqrt[3]{x}$	0,622	0,850	1,000	1,234	2,281
etäisyys $y$	0,387	0,723	1,0	1,523	5,203
$\sqrt{y}$	0,622	0,850	1,000	1,234	2,281

b)  $\sqrt{y} = \sqrt[3]{x} \parallel ( )^2$

$$y = \sqrt[3]{x}^2 \quad (y = x^{\frac{2}{3}})$$

c)  $x = 29,457$

$$y = \sqrt[3]{29,457}^2 = 9,5380 \dots \approx \underline{\underline{9,538 \text{ (maan et.)}}}$$

8. pizzoja 3

2 täytettä  $\swarrow$   $\searrow$  3 täytettä

$\binom{15}{2}$   $\binom{15}{3}$

a)  $3 \cdot \binom{15}{2} + 3 \cdot \binom{15}{3} = 1680$  eril. pizzoja

$$\frac{1680}{5} = 336$$

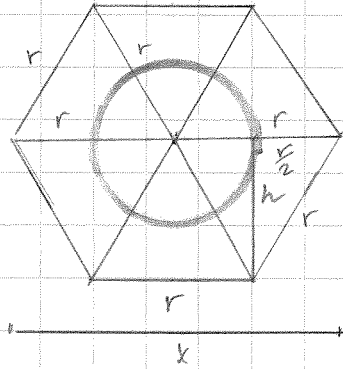
V: 336 viikkoa  
(yli 6 vuotta!)

b)  $\bar{x} = \frac{\binom{15}{2} \cdot 7,50 + \binom{15}{2} \cdot 8,50 + \binom{15}{2} \cdot 10,50 + \binom{15}{3} \cdot 8,50 + \binom{15}{3} \cdot 9,50 + \binom{15}{3} \cdot 11,5}{1680}$

$$= \frac{16205}{1680} = 9,645 \dots$$

V: 9,65 e

9.



a)  $x = \underline{2r}$

$$b) \quad h^2 + \left(\frac{r}{2}\right)^2 = r^2$$

$$h^2 = r^2 - \frac{r^2}{4}$$

$$h^2 = \frac{3}{4} r^2 \quad || \sqrt{\quad}$$

$$h = \frac{\sqrt{3}}{2} r$$

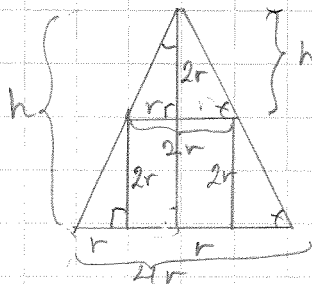
$$y = 2h = \underline{r\sqrt{3}}$$

c) 6 kolmiota  $6 \cdot \frac{1}{2} \cdot r \cdot h = 3 \cdot r \cdot \frac{\sqrt{3}}{2} r = \frac{3\sqrt{3}}{2} r^2$

ympyrä  $\pi \cdot \left(\frac{r}{2}\right)^2 = \pi \cdot \frac{r^2}{4} = \frac{\pi}{4} r^2$

$$A = \frac{3\sqrt{3}}{2} r^2 - \frac{\pi}{4} r^2 = \left(\frac{3\sqrt{3}}{2} - \frac{\pi}{4}\right) r^2 = \underline{\underline{\frac{6\sqrt{3} - \pi}{4} r^2 \approx 1,8r^2}}$$

10.



$h = 2r$  Suorakulmaisat kolmiot ovat  
yhdennuotoiset (kk).

(suora kulma ja yhteinen huippukulma,  
suhteessa  $\frac{r}{r} = 1$ .)

$\Rightarrow$  sama korkeus  $2r \Rightarrow$  kartion korkeus on  $4r$   
tai huippukolmio ja alkup. kolmio  $\sim$  (kk)

(huippukulma yht., kantakulmat samanlahtaisina)

$$\frac{h-2r}{h} = \frac{2r}{2r} \Rightarrow 2h - 4r = h$$

$$h = 4r$$

$$\frac{V_{\text{lierio}}}{V_{\text{kartio}}} = \frac{\pi \cdot r^2 \cdot 2r}{\frac{1}{3} \cdot \pi \cdot (2r)^2 \cdot 4r} = \frac{3 \cdot 2r^3}{4r^2 \cdot 4r} = \frac{3}{8} = 0,375$$

$$V: \underline{\underline{37,5\%}}$$

11. a) nainen  $y = 0,43x - 27$ ,  $y = 41 \text{ cm}$

$$0,43x - 27 = 41$$

$$0,43x = 68 \quad || : 0,43$$

$$x = 158,139\dots$$

$$V: \underline{\underline{158 \text{ cm}}}$$

b) mies  $y = 0,45x - 31$   $x = 175 \text{ cm}$ ,  $y = 42 \text{ cm}?$

$$y = 0,45 \cdot 175 - 31 = 47,75 \text{ selvästi}$$

suurempi kuin 42 cm, joten

$$V: \underline{\underline{\text{Ei ole}}}$$

12. 2004 6,4 mrd      2010 - 2004 = 6  
2010 6,8 mrd

MAL  
S 12

$$6,4 \cdot 10^9 \cdot x^6 = 6,8 \cdot 10^9$$

$$x^6 = \frac{6,8}{6,4} \quad // \sqrt[6]{\phantom{x}}$$

$$x = \sqrt[6]{\frac{6,8}{6,4}} = 1,01015\dots$$

$$6,8 \cdot 10^9 \cdot 1,01\dots^t > 10 \cdot 10^9$$

$$1,01\dots^t > \frac{10}{6,8} \quad // \lg$$

$$t \lg 1,01\dots > \lg 1,4705\dots \quad // : \lg 1,01\dots$$

$$t > 38,168\dots$$

$$2010 + 38,168\dots \approx 2048,2$$

V: Vuonna 2048

13. Karoliinan todellinen korkeus  
 $0,7 \cdot 0,022 \cdot 10\,000 = 154$

Petterin ens. puolen vuoden tod. korkeus

$$\frac{1}{2} \cdot 0,7 \cdot 0,0235 \cdot 10\,000 = 82,25$$

$$\text{sitten sijoitus on } 10\,000 + 82,25 = 10\,082,25$$

toinen puoliskoa

$$\frac{1}{2} \cdot 0,7 \cdot 0,02 \cdot 10\,082,25 = 70,575\dots \approx 70,58$$

Karoliinan sijoituksen arvo oli

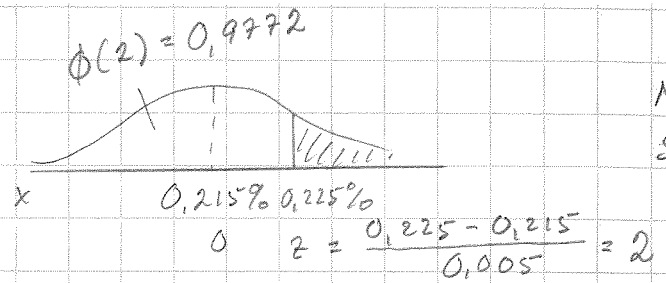
$$10\,000 + 154 = 10\,154 \text{ (e)}$$

Petterin

$$10\,082,25 + 70,58 = 10\,152,83 \text{ (e)} < 10\,154 \text{ (e)}$$

Sii Karoliinan sijoitus oli parempi,  
arvo 10154 e

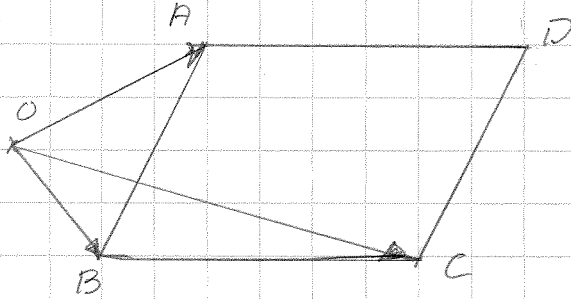
14.  $\bar{x} = 0,215\%$   
 $s = 0,005\%$



MAK  
S12

$P(x > 0,225\%)$   
 $= P(z > 2) = 1 - \phi(2) = 1 - 0,9772$   
 $= 0,0228 \approx \underline{\underline{2,3\%}}$

15.



$\vec{OA} = 4\vec{i} + 2\vec{j} + \vec{k}$   
 $\vec{OB} = 6\vec{i} + 5\vec{j} + 2\vec{k}$   
 $\vec{OC} = 7\vec{i} + 9\vec{j} + 3\vec{k}$

$\vec{OD} = \vec{OA} + \vec{AD} = \vec{OA} + \vec{BC} = \vec{OA} - \vec{OB} + \vec{OC}$   
 $= 4\vec{i} + 2\vec{j} + \vec{k} - (6\vec{i} + 5\vec{j} + 2\vec{k}) + 7\vec{i} + 9\vec{j} + 3\vec{k}$   
 $= (4 - 6 + 7)\vec{i} + (2 - 5 + 9)\vec{j} + (1 - 2 + 3)\vec{k}$   
 $\vec{OD} = \underline{\underline{5\vec{i} + 6\vec{j} + 2\vec{k}}}$

$\vec{AC} = -\vec{OA} + \vec{OC} = -4\vec{i} - 2\vec{j} - \vec{k} + 7\vec{i} + 9\vec{j} + 3\vec{k}$   
 $\vec{AC} = \underline{\underline{3\vec{i} + 7\vec{j} + 2\vec{k}}}$

$\vec{BD} = -\vec{OB} + \vec{OD} = -6\vec{i} - 5\vec{j} - 2\vec{k} + 5\vec{i} + 6\vec{j} + 2\vec{k}$   
 $\vec{BD} = \underline{\underline{-\vec{i} + \vec{j}}}$